SAMPLE CONTENT

Precise

MATHEMATICS

PART - 2

#itna hi kaafi hain

Std. XI
Science

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PRECISE

MATHEMATICS - II

Std. XI Sci. & Arts

Salient Features

- Written as per the latest textbook
- Exhaustive coverage of entire syllabus
- Covers all derivations and theorems
- Tentative marks allocation for all the problems
- The chapters include:
  - 'Precise Theory' for every topic
  - Solutions to all Exercises and Miscellaneous exercises given in the textbook
  - 'One Mark Questions' and 'Multiple choice questions' (MCQs)
- Includes Important Features for holistic learning:
  - **Smart Check** - **Important Formulae** - **Remember This**

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PREFACE

"Everything should be made as simple as possible, but not simpler." - Albert Einstein.

Inspired by this vision, we present **'Precise Mathematics – II: Std. XI'**, tailored to the latest Maharashtra State Board textbook. This compact yet comprehensive book aims to boost students' confidence and prepare them for the crucial Std. XI final exam, laying a solid foundation for their concepts.

Inside, you'll find **answers to all textbook exercises, including miscellaneous problems**. To aid in understanding, we've provided **precise theory** where needed, along with essential **theorems and their derivations**. A recap of all **important formulae** is included at the end of the book for quick revision.

We recognize that many problems can be tackled using various methods. That's why we've included an **'Alternate Method'** section to introduce students to different problem-solving approaches. To ensure accuracy, **'Smart Check'** helps students verify their answers effectively. Additionally, each chapter concludes with **'One Mark Questions'** and **'Multiple Choice Questions'**, along with their answers.
'Precise Mathematics – II: Std. XI' embodies our vision and achieves multiple goals: building concepts, developing problem-solving competence, and promoting self-study, all while encouraging cognitive thinking.

Refer to the flow chart on the adjacent page for an overview of the book's key features and how they are designed to enhance student learning.

We hope the book benefits the learner as we have envisioned.

Publisher
Edition: First

The journey to create a complete book is strewn with triumphs, failures and near misses. If you think we've nearly missed something or want to applaud us for our triumphs, we'd love to hear from you.
Please write to us on: mail@targetpublications.org

KEY FEATURES

Smart Check

Smart Check is a technique to verify the answers. This is our attempt to cross-check the accuracy of the answer. Smart check is indicated by ✓ symbol

One Mark Questions

These questions require very short solutions with direct application of mathematical concepts.

Multiple Choice Questions

Multiple Choice Questions include textual as well as additional MCQs.

Important Formulae

Important Formulae given at the end of the book include all of the key formulae in the chapter.
This is our attempt to offer students a handy tool to solve problems and ace the last minute revision.

CONTENTS

[Reference: Maharashtra State Board of Secondary and Higher Secondary Education, Pune - 04**]**

Smart check is indicated by ✓ symbol.

1 Complex Numbers

Contents and Concepts

- A Complex Number (C.N.)
- Algebra of C.N.
- Geometrical Representation of C.N.
- Polar and Exponential form of C.N.
- De Moivre's Theorem.

Chapter at a glance

1. Complex numbers:

i. If $z = x + iy$, where $x, y \in R$ and $i = \sqrt{-1}$ is called a complex number whose real part is x and imaginary part is y, i.e., Re (z) = x and Im (z) = y.
The complex number z is purely real if Im (z) = 0 and purely imaginary if Re (z) = 0.

ii. Integral powers of iota (i):
$i^2 = -1$
$i^3 = -i$
$i^4 = 1$
In general,
$$\left.\begin{array}{l} i^{4n} = 1, \quad i^{4n+1} = i \\ i^{4n+2} = -1, \ i^{4n+3} = -i \end{array}\right\} \text{ where } n \in N$$

2. Equality of two complex numbers:
The complex numbers $z_1 = a_1 + ib_1$ and $z_2 = a_2 + ib_2$ are equal iff $a_1 = a_2$ and $b_1 = b_2$ i.e., Re (z_1) = Re (z_2) and Im (z_1) = Im (z_2)

3. Conjugate of a complex number:
Conjugate of a complex number z = (a + ib) is defined as $\bar{z} = a - ib$.

4. Properties of Conjugate of a complex number:
If z_1, z_2, z_3 are complex numbers, then $\bar{z}$ is the mirror image of z along real axis

i. $\bar{\bar{z}} = z$

ii. $z + \bar{z} = 2\text{Re}(z)$

iii. $z - \bar{z} = 2i\,\text{Im}(z)$

iv. $z = \bar{z} \Leftrightarrow z$ is purely real.

v. $z + \bar{z} = 0 \Leftrightarrow z$ is purely imaginary.

vi. $z.\bar{z} = [\text{Re}(z)]^2 + [\text{Im}(z)]^2$

vii. $\overline{z_1 + z_2} = \bar{z}_1 + \bar{z}_2$

viii. $\overline{z_1 - z_2} = \bar{z}_1 - \bar{z}_2$

ix. $\overline{z_1 . z_2} = \bar{z}_1 . \bar{z}_2$

x. $\overline{\left(\frac{z_1}{z_2}\right)} = \frac{\bar{z}_1}{\bar{z}_2}, \ \bar{z}_2 \neq 0$

xi. $\overline{z^n} = \left(\bar{z}\right)^n$

xii. $z_1\bar{z}_2 + \bar{z}_1 z_2 = 2\text{Re}(\bar{z}_1 z_2) = 2\text{Re}(z_1\bar{z}_2)$

5. Algebra of complex numbers:

i. Addition of complex numbers:
The sum of two complex numbers is defined as
(a + ib) + (c + id) = (a + c) + i (b + d)
Thus, $\text{Re}(z_1 + z_2) = \text{Re}(z_1) + \text{Re}(z_2)$
and $\text{Im}(z_1 + z_2) = \text{Im}(z_1) + \text{Im}(z_2)$

Properties of addition of complex numbers:

a. Addition is commutative:
If z_1 and z_2 are two complex numbers, then
$z_1 + z_2 = z_2 + z_1$

b. Addition is associative:
Let z_1, z_2 and z_3 be three complex numbers, then
$(z_1 + z_2) + z_3 = z_1 + (z_2 + z_3)$

c. Existence of additive identity:
The complex number o = 0 + i0 is the identity element for addition i.e.,
$z + o = z = o + z \ \forall \ z \in C$.

d. Existence of additive inverse:
For any complex number
$z = a + ib, \ \exists - z = -a + i(-b)$ such that
$z + (-z) = o = (-z) + z$

$\therefore$ The complex number $-z = -a + i(-b)$ is called negative or additive inverse of z.

ii. Subtraction of complex numbers:
The subtraction of two complex numbers is defined as
(a + ib) − (c + id) = (a − c) + i (b − d)

iii. Multiplication of complex numbers:
The product of two complex numbers is defined as
(a + ib) (c + id) = (ac − bd) + i (ad + bc)

Properties of multiplication:

a. **Multiplication is commutative:**
If z_1, z_2 are two complex numbers, then $z_1 z_2 = z_2 z_1$.

b. **Multiplication is associative:**
Let z_1, z_2 and z_3 be three complex numbers, then $(z_1 z_2) z_3 = z_1(z_2 z_3)$.

c. **Existence of identity element for multiplication:**
The complex number $1 = 1 + i0$ is the identity element for multiplication i.e., for every complex number z, we have $z.1 = z = 1.z$

d. **Existence of multiplicative inverse:**
Corresponding to every non-zero complex number $z = a + ib$, $\exists$ a complex number $z_1 = x + iy$ such that $z.z_1 = 1 = z_1.z$
The multiplicative inverse of z is denoted by z^{-1} or $\frac{1}{z}$.

e. **Multiplication is distributive over addition:**
If z_1, z_2, z_3 are any three complex numbers, then

i. $z_1 (z_2 + z_3) = z_1 z_2 + z_1 z_3$ (Left distributive)

ii. $(z_2 + z_3) z_1 = z_2 z_1 + z_3 z_1$ (Right distributive)

iv. **Division of complex numbers:**
Let a + ib and c + id be any two complex numbers, where c + id is non-zero, then division is defined as

$$\frac{a+ib}{c+id} = \frac{a+ib}{c+id} \times \frac{c-id}{c-id}$$
$$= \frac{ac+bd}{c^2+d^2} + i\,\frac{bc-ad}{c^2+d^2}$$

6. **Square root of a Complex number:**
Let $x + iy$ be a square root of a + ib.

$\therefore$ $x + iy = \sqrt{a+ib}$
Squaring both sides, we get
$(x + iy)^2 = a + ib$

$\therefore$ $x^2 - y^2 + 2xyi = a + ib$
Equating real and imaginary parts, we get $x^2 - y^2 = a$ and $2xy = b$
Solving these equations, we can find x and y then $x + iy$ will be the required square root of a + ib.

7. **Fundamental theorem of Algebra:**
Solution of a quadratic equation in complex number system:

i. Consider the quadratic equation $ax^2 + bx + c = 0$, where a, b, c $\in$ R and a $\neq$ 0
On solving this quadratic equation, we get
$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$
$\therefore$ $x = \frac{-b+\sqrt{b^2-4ac}}{2a}$ and $x = \frac{-b-\sqrt{b^2-4ac}}{2a}$
are the roots of the equation $ax^2 + bx + c = 0$.

ii. The expression $(b^2 - 4ac)$ is called the discriminant (D).
If D < 0, then the roots of the given quadratic equation are complex.

iii. If p + iq is the root of the equation $ax^2 + bx + c = 0$, then p − iq is also a another root of the given quadratic equation

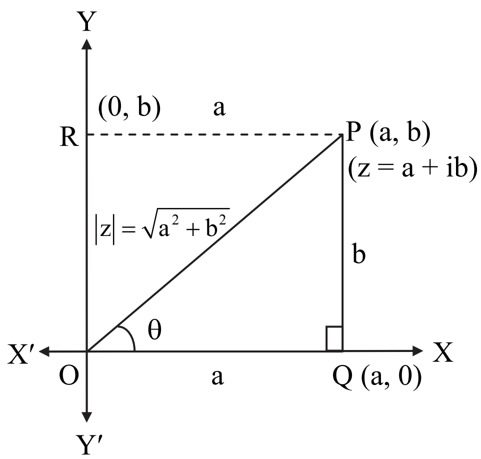
$\therefore$ complex roots occur in conjugate pairs.

iv. If D = 0, then the roots of the given quadratic equation are real and equal.

8. **Modulus of a complex number:**
Modulus of a complex number z = a + ib denoted by $|z|$ is defined as $|z| = \sqrt{a^2+b^2}$ or $|z| = \sqrt{(\text{Re}(z))^2 + (\text{Im}(z))^2}$.

9. **Argument of a complex number:**
If $z \neq 0$, the argument (amplitude) θ of z is defined by two equations:
$\cos\theta = \frac{a}{|z|}$; $\sin\theta = \frac{b}{|z|}$
So $\arg z = \theta = \tan^{-1}\left(\frac{b}{a}\right)$, $0 \le \theta < 2\pi$
It is denoted by arg z or amp z.

10. **Geometrical Meaning of Modulus and Argument(Argand's Diagram):**

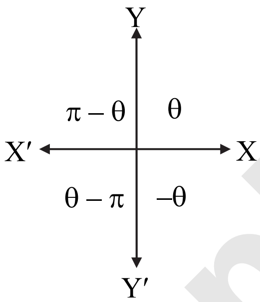


i. **Modulus of z** (denoted by $|z|$) : The length of the line segment OP is called $|z|$
$\Rightarrow |z| = OP = \sqrt{a^2+b^2}$

ii. **Argument or Amplitude of z** (denoted by arg (z) or amp (z)) :
The angle θ which OP makes with +ve direction of X–axis in anticlockwise direction is called arg (z).
From the above figure,
$\cos\theta = \dfrac{a}{\sqrt{a^2+b^2}}$, $\sin\theta = \dfrac{b}{\sqrt{a^2+b^2}}$,
$\tan\theta = \dfrac{b}{a}$
$\Rightarrow \boldsymbol{\theta = \tan^{-1}\left(\dfrac{b}{a}\right)}$

iii. **Principal arg (z):** The argument θ which satisfies the inequality $-\pi < \theta \le \pi$ is known as the principal argument of z.
This is denoted by Pr. arg (z) or Arg (z).

iv. **Argument of z in different quadrants/axes:**
Let z = a + ib = (a, b) and $\tan^{-1}\left|\dfrac{b}{a}\right| = \alpha$.
Then, arg (z) = $\tan^{-1}\left(\dfrac{b}{a}\right)$ always gives the principal value. It depends upon the quadrant in which the point (a, b) lies.



a. Arg (z) = $\tan^{-1}\left|\dfrac{b}{a}\right|$, when z lies in 1st quadrant.

b. Arg (z) = $\pi - \tan^{-1}\left|\dfrac{b}{a}\right|$, when z lies in 2nd quadrant.

c. Arg (z) = $\tan^{-1}\left|\dfrac{b}{a}\right| - \pi$ or $\pi + \tan^{-1}\left|\dfrac{b}{a}\right|$, when z lies in 3rd quadrant.

d. Arg (z) = $-\tan^{-1}\left|\dfrac{b}{a}\right|$ or $2\pi - \tan^{-1}\left|\dfrac{b}{a}\right|$ when z lies in 4th quadrant.

11. Properties of modulus of complex numbers:
If z_1, z_2, z_3 are complex numbers, then

i. $|z| = 0 \Leftrightarrow z = 0$
i.e., Re (z) = Im (z) = 0

ii. $|z| = |\bar{z}| = |-z| = |-\bar{z}|$

iii. $-|z| \le \text{Re}(z) \le |z|$; $-|z| \le \text{Im}(z) \le |z|$

iv. $z\bar{z} = |z|^2$

v. $|z_1 z_2| = |z_1|\,|z_2|$

vi. $\left|\dfrac{z_1}{z_2}\right| = \dfrac{|z_1|}{|z_2|}$, $z_2 \neq 0$

vii. $|z_1 + z_2|^2 = |z_1|^2 + |z_2|^2 + 2\text{Re}(z_1\bar{z}_2)$

viii. $|z_1 - z_2|^2 = |z_1|^2 + |z_2|^2 - 2\text{Re}(z_1\bar{z}_2)$

ix. $|z_1 + z_2|^2 + |z_1 - z_2|^2 = 2(|z_1|^2 + |z_2|^2)$

x. $|az_1 - bz_2|^2 + |bz_1 + az_2|^2$
$= (a^2 + b^2)(|z_1|^2 + |z_2|^2)$, where a, b ∈ R

xi. $|z_1 \pm z_2| \le |z_1| + |z_2|$

xii. $|z_1 \pm z_2| \ge \big||z_1| - |z_2|\big|$

xiii. $|z^n| = |z|^n$

xiv. $\big||z_1| - |z_2|\big| \le |z_1 + z_2| \le |z_1| + |z_2|$

xv. $|z_1 \pm z_2|^2 = (z_1 \pm z_2)(\bar{z}_1 \pm \bar{z}_2)$

xvi. $z_1\bar{z}_2 + \bar{z}_1 z_2 = 2|z_1||z_2|\cos(\theta_1 - \theta_2)$,
where θ_1 = arg (z_1) and θ_2 = arg (z_2)

12. Properties of arg (z):

i. arg (any +ve real no.) = 0,
arg (any +ve imaginary no.) = $\dfrac{\pi}{2}$

ii. arg (any –ve real no.) = π,
arg (any –ve imaginary no.) = $-\dfrac{\pi}{2}$

iii. arg (z_1. z_2) = arg (z_1) + arg (z_2)

iv. $\arg\left(\dfrac{z_1}{z_2}\right)$ = arg (z_1) – arg (z_2)

v. $\arg(\bar{z}) = -\arg(z) = \arg\left(\dfrac{1}{z}\right)$

vi. arg (+ z) = π ± arg (z) and
arg (– z) = arg z ± π

vii. $\arg(z) + \arg(\bar{z}) = 0$

13. Polar form of a complex number
The polar form of a complex number $z = x + iy$ is z = r (cos θ + i sin θ), where
$r = \sqrt{x^2 + y^2} = |z|$ and $x = r\cos\theta$, $y = r\sin\theta$.

14. Euler's form or Exponential form:
$e^{i\theta} = \cos\theta + i\sin\theta = \text{cis}\,\theta$

15. DeMoivre's Theorem:

i. If n ∈ Z (set of integers), then
$(\cos\theta + i\sin\theta)^n = \cos n\theta + i\sin n\theta$

ii. If n ∈ Q (set of rational numbers),
then $(\cos\theta + i\sin\theta)^n = \cos n\theta + i\sin n\theta$

iii. $(\cos\theta + i\sin\theta)^{-n} = \cos n\theta - i\sin n\theta$

iv. $(\cos\theta - i\sin\theta)^n = \cos n\theta - i\sin n\theta$

v. $\dfrac{1}{(\cos\theta + i\sin\theta)^1} = \cos\theta - i\sin\theta$

vi. $(\cos\theta - i\sin\theta)^{-n} = \cos n\theta + i\sin n\theta$

16. Cube roots of unity:

$x^3 = 1$

$\Rightarrow x^3 - 1 = 0$

$\Rightarrow (x - 1)(1 + x + x^2) = 0$

$\Rightarrow x = 1$ or $1 + x + x^2 = 0$

$\Rightarrow x = 1$ or $x = \dfrac{-1 \pm \sqrt{(1)^2 - 4(1)(1)}}{2(1)}$

$\Rightarrow x = 1$ or $x = \dfrac{-1}{2} \pm \dfrac{\sqrt{-3}}{2}$

$\Rightarrow x = 1$ or $x = \dfrac{-1}{2} \pm \dfrac{i\sqrt{3}}{2}$

$\therefore$ The cube roots of unity are 1, $\dfrac{-1}{2} + \dfrac{i\sqrt{3}}{2}$ and $\dfrac{-1}{2} - \dfrac{i\sqrt{3}}{2}$.

Properties of cube roots of unity:

If ω is a complex cube root of unity, then

i. $\omega^3 = 1$

ii. $1 + \omega + \omega^2 = 0$

where, $\omega = \dfrac{-1 + i\sqrt{3}}{2}$ and $\omega^2 = \dfrac{-1 - i\sqrt{3}}{2}$

iii. $\omega^2 = \dfrac{1}{\omega}$ and $\omega = \dfrac{1}{\omega^2}$

iv. $\omega^{3n+1} = \omega$ and $\omega^{3n+2} = \omega^2$

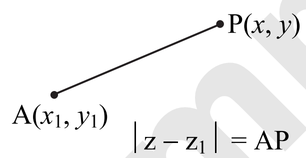
v. $\overline{\omega} = \omega^2$

vi. $(\overline{\omega})^2 = \omega$

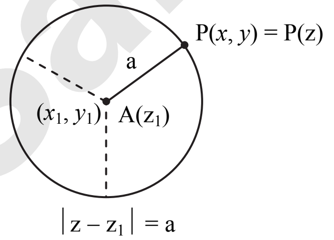
17. Set of points in complex plane:

If $z = x + iy$ represents the variable point $P(x, y)$ and $z_1 = x_1 + iy$ represents the fixed point $A(x_1, y_1)$, then

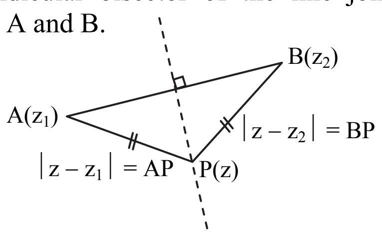
i. $|z - z_1|$ represents the length of AP



ii. $|z - z_1| = a$ represents the circle with centre $A(x_1, y_1)$ and radius a.



iii. $|z - z_1| = |z - z_2|$ represents the perpendicular bisector of the line joining the points A and B.



Let's Study

Introduction

A linear equation in x is in the form $ax + b = 0$ having a real root $\dfrac{-b}{a}$. Solution of a quadratic equation is obtained by factorization.

But every quadratic equation is not factorizable such as $x^2 + 1 = 0$.

Now, $x^2 + 1$ has no factors in the set of real numbers.

Also, $x^2 = -1$ is not possible in the set of real numbers, as squares of real numbers are non-negative.

Inspite of the facts mentioned, the solution set of equation $x^2 + 1 = 0$ is $x = \pm\sqrt{-1}$, where $\sqrt{-1}$ is called imaginary unit and it is denoted by i.

i.e., $i = \sqrt{-1}$

$\therefore \quad i^2 = -1$

In general, $x = \pm\sqrt{a}\, i$ is the solution of equation $x^2 + a = 0$, where a is a positive real number.

Thus i is an imaginary number.

Now, consider the equation $x^2 - 6x + 13 = 0$.

$\therefore \quad x^2 - 6x + 9 = -4$

$\therefore \quad (x - 3)^2 = 4i^2$

$\therefore \quad x - 3 = \pm 2i$

$\therefore \quad x = 3 \pm 2i$

$\therefore \quad x = 3 + 2i$ or $x = 3 - 2i$

Hence the equation $x^2 - 6x + 13 = 0$ has two solutions $3 + 2i$ and $3 - 2i$, which are not real numbers. These numbers are called *complex numbers.*

Complex Numbers

Imaginary number:

A number of form bi, where $b \in R$, $b \neq 0$, $i = \sqrt{-1}$ is called an imaginary number.

Example:

$\sqrt{-36} = 6i$, 3i, $-\dfrac{4}{9}i$ etc.

Note:

The number i satisfies following properties,

i. $i \times 0 = 0$

ii. If $a \in R$, then $\sqrt{-a^2} = \sqrt{i^2a^2} = \pm ia$.

iii. If $a, b \in R$, and $ai = bi$, then $a = b$.

Complex number:

Definition:

A number of the type $a + ib$ or $a + bi$, where a and b are real numbers and $i = \sqrt{-1}$ is called a complex number.

i. In a complex number $a + ib$, a is called the real part and b is called the imaginary part of the complex number $a + ib$.

ii. Note that real part and imaginary part of complex number are real numbers.
The complex number is denoted by z.

∴ $z = a + ib$
where real part denoted by Re(z) or R(z) and Imaginary part denoted by Im(z) or I(z)

∴ $z = \text{Re}(z) = \text{R}(z) = a$

∴ $\text{Im}(z) = \text{I}(z) = b$

Example:
If $z = 2 + 3i$ is a complex number, then
$\text{Re}(z) = 2$ and $\text{Im}(z) = 3$

Note:
i. A complex number whose real part is zero is called a purely imaginary number. Such a number is of the form $z = 0 + ib = ib$.
ii. A complex number whose imaginary part is zero is a real number.
$z = a + 0i = a$, is a real number.
iii. A complex number whose both real and imaginary parts are zero is the zero complex number. $0 = 0 + 0i$.
iv. The set R of real numbers is a subset of the set C of complex numbers.
v. The real part and imaginary part cannot be combined to form single term. E.g. $5 + 2i \neq 3i$.

Algebra of complex numbers

1. Equality of two complex numbers:
Two complex numbers $z_1 = a + bi$ and $z_2 = c + id$ are said to be equal if their corresponding real and imaginary number parts are equal.
Two complex numbers $a + ib$ and $c + id$ are said to be equal if $a = c$ and $b = d$
i.e., $a + ib = c + id$, if $a = c$ and $b = d$

2. Conjugate of a complex number
If $a + ib$ is a complex number, then $a - ib$ is the conjugate complex number of $a + ib$. If $z = a + ib$ then its conjugate complex number is denoted by $\bar{z}$.

∴ $\bar{z} = a - ib$

Example:

Complex numbers	Conjugate complex numbers
$3 + 2i$	$3 - 2i$
$4 - \sqrt{5}\,i$	$4 + \sqrt{5}\,i$
$2i - 3$	$-3 - 2i$
$\cos\theta + i\sin\theta$	$\cos\theta - i\sin\theta$

Properties of conjugate of a complex number
i. $\overline{(\bar{z})} = z$
ii. $z + \bar{z} = 2\,\text{Re}(z)$
iii. $z - \bar{z} = 2i.\,\text{Im}(z)$
iv. $z = \bar{z}$
∴ z is real
v. Let $z \neq 0$.
$\bar{z} + z = 0$
∴ z is purely imaginary.
vi. $\overline{z_1 + z_2} = \bar{z}_1 + \bar{z}_2$
vii. $\overline{z_1 - z_2} = \bar{z}_1 - \bar{z}_2$
viii. $\overline{z_1 z_2} = \bar{z}_1 \cdot \bar{z}_2$
ix. $\overline{(z_1 z_2 z_3 \dots z_n)} = \bar{z}_1 \cdot \bar{z}_2 \cdot \bar{z}_3 \dots \bar{z}_n$
x. $\overline{\left(\dfrac{z_1}{z_2}\right)} = \dfrac{\bar{z}_1}{\bar{z}_2}$, $z_2 \neq 0$

3. Addition of complex numbers:
If $z_1 = a_1 + ib_1$ and $z_2 = a_2 + ib_2$ are two complex numbers, then their sum is $z_1 + z_2$ and is defined as $z_1 + z_2 = (a_1 + ib_1) + (a_2 + ib_2)$
$= (a_1 + a_2) + i(b_1 + b_2)$

∴ $\text{Re}(z_1 + z_2) = \text{Re}(z_1) + \text{Re}(z_2)$
and $\text{Im}(z_1 + z_2) = \text{Im}(z_1) + \text{Im}(z_2)$
Thus $z_1 + z_2$ is a complex number.

Example:
i. $(3 - 7i) + (5 + 3i) = (3 + 5) + i(-7 + 3)$
$= 8 - 4i$
ii. $(-2 + 5i) + (3 - 7i) = (-2 + 3) + i(5 - 7)$
$= 1 - 2i$

Properties of addition:
If z_1, z_2, z_3 are complex numbers, then
i. $z_1 + z_2 = z_2 + z_1$ (commutative)
ii. $z_1 + (z_2 + z_3) = (z_1 + z_2) + z_3$ (associative)
iii. $z_1 + 0 = 0 + z_1 = z_1$ (identity)
iv. $z_1 + \bar{z}_1 = 2\text{Re}(z_1)$
v. $\overline{z_1 + z_2} = \bar{z}_1 + \bar{z}_2$

4. Subtraction of complex numbers:
If $z_1 = a_1 + ib_1$ and $z_2 = a_2 + ib_2$ are two complex numbers, then their subtraction is $z_1 - z_2$ and is defined as
$z_1 - z_2 = (a_1 + ib_1) - (a_2 + ib_2)$
$= (a_1 - a_2) + i(b_1 - b_2)$
Thus $z_1 - z_2$ is a complex number.

Example:
i. $(4 + i) - (2 - 3i) = (4 - 2) + (1 + 3)i$
$= 2 + 4i$
ii. $(5 + 13i) - (4 + 7i) = (5 - 4) + (13 - 7)\,i$
$= 1 + 6i$

5. Scalar multiplication
If $z = a + ib$ is any complex number, then for every real number k, define $kz = ka + i(kb)$

Example:
i. If $z = 7 + 3i$, then
$5z = 5(7 + 3i) = 35 + 15i$

ii. $z_1 = 3 - 4i$ and $z_2 = 10 - 9i$, then

$2z_1 + 5z_2 = 2(3 - 4i) + 5(10 - 9i)$
$= 6 - 8i + 50 - 45i$
$= 56 - 53i$

Note:

$0.z = 0(a + ib) = 0 + 0i = 0$

6. Multiplication of complex numbers:

If $z_1 = a_1 + ib_1$ and $z_2 = a_2 + ib_2$ are any two complex numbers, then their product is $z_1.z_2$ and is defined as

$$z_1.z_2 = (a_1 + ib_1)(a_2 + ib_2)$$
$$= a_1a_2 + i(a_1b_2) + i(b_1a_2) + i^2(b_1b_2)$$
$$= a_1a_2 + i(a_1b_2 + b_1a_2) - b_1b_2 \quad \ldots[\because i^2 = -1]$$
$$= (a_1a_2 - b_1b_2) + i(a_1b_2 + b_1a_2)$$

Thus product $z_1.z_2$ is a complex number.

Example:

i. $(1 + i)(2 - 3i) = 2 + 2i - 3i - 3i^2$
$= 2 - i + 3 \quad \ldots[\because i^2 = -1]$
$= 5 - i$

ii. $(2 + i)(2 - i) = (2)^2 - (i^2)$
$= 4 + 1$
$= 5$

Properties of multiplication:

i. $z_1.z_2 = z_2.z_1$ (commutative)

ii. $(z_1.z_2).z_3 = z_1.(z_2.z_3)$ (associative)

iii. $(z_1.1) = 1.z_1 = z_1$ (identity)

iv. $\left(\overline{z_1.z_2}\right) = \overline{z_1}\,.\,\overline{z_2}$

v. If $z = a + ib$, then $z.\bar{z} = a^2 + b^2$

Try This

1. Verify: $z + \bar{z} = 2\text{Re}(z)$ *(Textbook page no. 3)*

Solution:

Let $z = a + bi$

$\therefore \bar{z} = a - ib$

$z + \bar{z} = a + bi + a - ib$
$= 2a$, which is a real part of z

$\therefore z + \bar{z} = 2\text{Re}(z)$

2. Verify: $z - \bar{z} = 2\text{Im}(z)$ *(Textbook page no. 3)*

Solution:

Let $z = a + bi$

$\therefore \bar{z} = a - ib$

$z - \bar{z} = a + ib - a + ib$
$= 2ib$, which is a imaginary part of z

$\therefore z - \bar{z} = 2\text{Im}(z)$

3. Verify: $\left(\overline{z_1 \cdot z_2}\right) = \bar{z}_1 \cdot \bar{z}_2$ *(Textbook page no. 3)*

Solution:

Let $z_1 = a + ib$ and $z_2 = c + id$

$\therefore \bar{z}_1 = a - ib$ and $\bar{z}_2 = c - id$

$z_1.z_2 = (a + ib)(c + id)$
$= ac + adi + bci - bd$
$= (ac - bd) + (ad + bc)i$

$\therefore \overline{z_1 \cdot z_2} = (ac - bd) - (ad + bc)i \quad$ ...(i)

$\bar{z}_1 \cdot \bar{z}_2 = (a - ib)(c - id)$
$= ac - adi - bci - bd$
$= (ac - bd) - (ad + bc)i \quad$ ...(ii)

From (i) and (ii), we get

$\overline{z_1 \cdot z_2} = \bar{z}_1 \cdot \bar{z}_2$

7. Powers of i:

Consider i^n, where n is a positive integer and $n > 4$.

Now divide n by 4 and let the quotient be m and the remainder obtained be 'r'.

$n = 4m + r$, where $0 \le r < 4$

$\therefore i^n = i^{(4m + r)}$

$\therefore i^n = (i^4)^m.i^r$

$\therefore i^n = i^r \quad \ldots.[\because i^4 = 1]$

Example:

$i^{82} = (i^4)^{20}.i^2 = (1)^{20}.i^2 = -1$

In general,

$$\left.\begin{array}{ll} i^{4n} = 1, & i^{4n+1} = i \\ i^{4n+2} = -1, & i^{4n+3} = -i \end{array}\right\} \text{ where } n \in N$$

8. Division of complex numbers:

Let $a + ib$ and $c + id$ be any two complex numbers, where $c + id$ is non-zero, then division is defined as

$$\frac{a + ib}{c + id} = \frac{a + ib}{c + id} \times \frac{c - id}{c - id}$$
$$= \frac{ac - adi + cbi - i^2bd}{c^2 - (id)^2}$$
$$= \frac{ac - adi + cbi - (-1)bd}{c^2 - i^2d^2}$$
$$= \frac{ac - adi + cbi + bd}{c^2 - (-1)d^2}$$
$$= \frac{ac + bd + (bc - ad)i}{c^2 + d^2}$$
$$= \frac{ac + bd}{c^2 + d^2} + i\frac{bc - ad}{c^2 + d^2}$$

Example:

If $z_1 = 2 + 3i$ and $z_2 = 1 + 2i$, then

$$\frac{z_1}{z_2} = \frac{2 + 3i}{1 + 2i} \times \frac{1 - 2i}{1 - 2i} = \frac{8}{5} - \frac{1}{5}i$$

Properties of division:

i. $\frac{1}{i} = \frac{1}{i} \times \frac{i}{i} = \frac{i}{-1} = -i$

ii. $\frac{1}{a + ib} = \frac{a - ib}{a^2 + b^2}$

Remember This

$i = \sqrt{-1}$
$i^2 = -1$
$i^3 = i^2\, i = (-1)\, i = -i$
$i^4 = (i^2)^2 = (-1)^2 = 1$
$i^5 = i^4.\, i = (1)^4 \times i = i$
$i^6 = (i^2)^3 = (-1)^3 = -1$ and so on
$\frac{1}{i} = -i$

Exercise 1.1

1. Simplify: **[2 Marks Each]**

i. $i\sqrt{16} + 3i\sqrt{25} + i\sqrt{36} - 25i$

ii. $4\sqrt{4}i + 5\sqrt{9}i - 3\sqrt{16}i$

Solution:

i. $i\sqrt{16} + 3i\sqrt{25} + i\sqrt{36} - 25i$
$= 4i + 3(5i) + 6i - 25i$
$= 25i - 25i$
$= 0$

ii. $4\sqrt{4}i + 5\sqrt{9}i - 3\sqrt{16}i$
$= 4(2i) + 5(3i) - 3(4i)$
$= 8i + 15i - 12i$
$= 11i$

2. Write the conjugates of the following complex numbers: **[1 Mark Each]**

i. $3 + i$ **ii.** $3 - i$
iii. $-\sqrt{5} - \sqrt{7}\, i$ **iv.** $-\sqrt{5}\, i$
v. $5i$ **vi.** $\sqrt{5} - i$
vii. $\sqrt{2} + \sqrt{3}\, i$ **viii.** $\cos\theta + i \sin\theta$

Solution:

i. Conjugate of (3 + i) is (3 − i).

ii. Conjugate of (3 − i) is (3 + i) .

iii. Conjugate of $\left(-\sqrt{5} - \sqrt{7}i\right)$ is $\left(-\sqrt{5} + \sqrt{7}i\right)$.

iv. Conjugate of $\left(-\sqrt{5}i\right)$ is $\sqrt{5}i$.

v. Conjugate of (5i) is (−5i).

vi. Conjugate of $\left(\sqrt{5} - i\right)$ is $\left(\sqrt{5} + i\right)$.

vii. Conjugate of ($\sqrt{2} + \sqrt{3}$ i) is ($\sqrt{2} - \sqrt{3}$ i).

viii. Conjugate of $(\cos\theta + i \sin\theta)$ is $(\cos\theta - i \sin\theta)$.

3. Find a and b if **[2 Marks Each]**

i. $a + 2b + 2ai = 4 + 6i$

ii. $(a - b) + (a + b)i = a + 5i$

iii. $(a + b)(2 + i) = b + 1 + (10 + 2a)i$

iv. $abi = 3a - b + 12i$

v. $\frac{1}{a+ib} = 3 - 2i$

vi. $(a + ib)(1 + i) = 2 + i$

Solution:

i. $a + 2b + 2ai = 4 + 6i$
Equating real and imaginary parts, we get
$a + 2b = 4$ …(i)
$2a = 6$ …(ii)
∴ $a = 3$
Substituting, a = 3 in (i), we get
$3 + 2b = 4$
∴ $b = \frac{1}{2}$
∴ $a = 3$ and $b = \frac{1}{2}$

Smart Check

For $a = 3$ and $b = \frac{1}{2}$:
Consider, L.H.S. $= a + 2b + 2ai$
$= 3 + 2\left(\frac{1}{2}\right) + 2(3)i$
$= 4 + 6i =$ R.H.S.

ii. $(a - b) + (a + b)i = a + 5i$
Equating real and imaginary parts, we get
$a - b = a$ …(i)
$a + b = 5$ …(ii)
From (i), b = 0
Substituting b = 0 in (ii), we get
$a + 0 = 5$
∴ $a = 5$
∴ $a = 5$ and $b = 0$

iii. $(a + b)(2 + i) = b + 1 + (10 + 2a)i$
∴ $2(a + b) + (a + b)i = (b + 1) + (10 + 2a)i$
Equating real and imaginary parts, we get
$2(a + b) = b + 1$
∴ $2a + b = 1$ …(i)
and $a + b = 10 + 2a$
$-a + b = 10$ …(ii)

Page no. **8** to **34** are purposely left blank.

To see complete chapter buy **Target Notes** or **Target E-Notes**

iv. Let $z = -2\sqrt{3} - 2i$

$\therefore$ $a = -2\sqrt{3}$, $b = -2$, i.e. $a < 0, b < 0$

$\therefore$ $|z| = \sqrt{a^2+b^2} = \sqrt{\left(-2\sqrt{3}\right)^2 + (-2)^2} = \sqrt{12+4}$

$= 4$

Here, $(-2\sqrt{3}, -2)$ lies in 3rd quadrant.

$\therefore$ amp (z) $= \tan^{-1}\left(\frac{b}{a}\right) - \pi = \tan^{-1}\left(\frac{-2}{-2\sqrt{3}}\right) - \pi$

$= \tan^{-1}\left(\frac{1}{\sqrt{3}}\right) - \pi$

$= \frac{\pi}{6} - \pi = \frac{-5\pi}{6}$

$z^5 = \left(-2\sqrt{3} - 2i\right)^5$

$= \left[4\left(\cos\frac{-5\pi}{6} + i\sin\frac{-5\pi}{6}\right)\right]^5$

$= 1024\left(\cos\frac{-25\pi}{6} + i\sin\frac{-25\pi}{6}\right)$

…[$\because$ $(\cos\theta + i\sin\theta)^n = (\cos n\theta + i\sin n\theta)$]

$= 1024\left(\cos\frac{25\pi}{6} - i\sin\frac{25\pi}{6}\right)$

$= 1024\left[\cos\left(4\pi + \frac{\pi}{6}\right) - i\sin\left(4\pi + \frac{\pi}{6}\right)\right]$

$= 1024\left(\cos\frac{\pi}{6} - i\sin\frac{\pi}{6}\right)$

$= 1024\left[\frac{\sqrt{3}}{2} - \frac{1}{2}i\right]$

$= 512\sqrt{3} - 512i$

[**Note:** Answer given in the textbook is '$512\sqrt{3} + 512i$'. However, as per our calculation it is $512\sqrt{3} - 512i$.]

Miscellaneous Exercise – 1

I. Select the correct answer from the given alternatives. [2 Marks Each]

1. If n is an odd positive integer, then the value of $1 + (i)^{2n} + (i)^{4n} + (i)^{6n}$ is:
(A) $-4i$ (B) 0
(C) $4i$ (D) 4

2. The value of $\frac{i^{592}+i^{590}+i^{588}+i^{586}+i^{584}}{i^{582}+i^{580}+i^{578}+i^{576}+i^{574}}$ is equal to:
(A) -2 (B) 1
(C) 0 (D) -1

3. $\left(\sqrt{3}i\right)\left(\sqrt{6}i\right)$ is equal to
(A) $-3\sqrt{2}$ (B) $3\sqrt{2}$
(C) $3\sqrt{2}i$ (D) $-3\sqrt{2}i$

4. If ω is a complex cube root of unity, then the value of $\omega^{99} + \omega^{100} + \omega^{101}$ is:
(A) -1 (B) 1 (C) 0 (D) 3

5. If $z = r(\cos\theta + i\sin\theta)$, then the value of $\frac{z}{\bar{z}} + \frac{\bar{z}}{z}$ is
(A) $\cos 2\theta$ (B) $2\cos 2\theta$
(C) $2\cos\theta$ (D) $2\sin\theta$

6. If $\omega(\neq 1)$ is a cube root of unity and $(1+\omega)^7 = A + B\omega$, then A and B are respectively the numbers
(A) 0,1 (B) 1, 1
(C) 1, 0 (D) -1, 1

7. The modulus and argument of $(1 + i\sqrt{3})^8$ are respectively
(A) 2 and $\frac{2\pi}{3}$ (B) 256 and $\frac{2\pi}{3}$
(C) 256 and $\frac{4\pi}{3}$ (D) 64 and $\frac{4\pi}{3}$

[**Note:** Option (C) has been modified.]

8. If arg (z) = θ, then arg $\overline{(z)}$ =
(A) $-\theta$ (B) θ
(C) $\pi - \theta$ (D) $\pi + \theta$

9. If $-1 + \sqrt{3}i = re^{i\theta}$, then θ = ________.
(A) $-\frac{2\pi}{3}$ (B) $\frac{\pi}{3}$ (C) $-\frac{\pi}{3}$ (D) $\frac{2\pi}{3}$

10. If $z = x + iy$ and $|z - zi| = 1$, then
(A) z lies on X-axis
(B) z lies on Y-axis
(C) z lies on a rectangle
(D) z lies on a circle

Answers:

1.	(B)	2.	(D)	3.	(A)	4.	(C)
5.	(B)	6.	(B)	7.	(B)	8.	(A)
9.	(D)	10.	(D)				

Hints:

1. $1 + (i^2)^n + (i^4)^n + (i^2)^{3n}$
$= 1 - 1 + 1 - 1$ …(n odd positive integer)
$= 0$

2. $\frac{i^{592}+i^{590}+i^{588}+i^{586}+i^{584}}{i^{582}+i^{580}+i^{578}+i^{576}+i^{574}}$
$= \frac{i^{584}\left[i^8+i^6+i^4+i^2+1\right]}{i^{574}\left[i^8+i^6+i^4+i^2+1\right]} = i^{10} = (i^2)^5$
$= (-1)^5 = -1$

3. $\left(\sqrt{3}i\right)\left(\sqrt{6}i\right) = 3\sqrt{2}(-1) = -3\sqrt{2}$

4. $\omega^{99} + \omega^{100} + \omega^{101} = \omega^{99}(1 + \omega + \omega^2)$
$= \omega^{99}(0) = 0$

5. $\frac{z}{\bar{z}} = \frac{r(\cos\theta + i\sin\theta)}{r(\cos\theta - i\sin\theta)} = \frac{\cos\theta + i\sin\theta}{\cos(-\theta) + i\sin(-\theta)}$

$= (\cos\theta + i\sin\theta)(\cos(-\theta) + i\sin(-\theta))^{-1}$

$= (\cos\theta + i\sin\theta)(\cos\theta + i\sin\theta)$ …[De Moivre's Theorem]

$= (\cos\theta + i\sin\theta)^2$

$= \cos 2\theta + i\sin 2\theta$ …[De Moivre's Theorem]

∵ $\overline{\left(\frac{z}{\bar{z}}\right)} = \frac{\bar{z}}{z}$

∴ $\frac{z}{\bar{z}} + \frac{\bar{z}}{z} = d + \bar{d}$ …where $d = \frac{z}{\bar{z}}$

$= \cos 2\theta + i\sin 2\theta + \cos 2\theta - i\sin 2\theta$

$= 2\cos 2\theta$

---

6. $(1+\omega)^7 = (-\omega^2)^7 = -\omega^{14}$

$= -\omega^2(\omega^3)^4$

$= -\omega^2 = 1 + \omega$

A = 1, B = 1

---

7. Let $z = (1 + i\sqrt{3})^8 = [r(\cos\theta + i\sin\theta)]^8$

$r\cos\theta = 1$, $r\sin\theta = \sqrt{3}$,

$r = \sqrt{1+3} = 2$

∴ $\cos\theta = \frac{1}{2}$, $\sin\theta = \frac{\sqrt{3}}{2}$

∴ $\arg z = \frac{\pi}{3}$

$z = \left[2\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)\right]^8$

$= 2^8\left(\cos\frac{8\pi}{3} + i\sin\frac{8\pi}{3}\right)$

$= 256\left[\cos\left(2\pi + \frac{2\pi}{3}\right) + i\sin\left(2\pi + \frac{2\pi}{3}\right)\right]$

$= 256\left[\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right]$

---

8. Let $z = re^{i\theta}$, then $\bar{z} = re^{-i\theta}$

∴ $\arg \bar{z} = -\theta$.

---

9. $re^{i\theta} = -1 + i\sqrt{3}$

$= 2\left(\frac{-1}{2} + i\frac{\sqrt{3}}{2}\right)$ …$\left[a = \frac{-1}{2}, b = \frac{\sqrt{3}}{2}\right]$

$= 2\left[\cos\left(\pi - \frac{\pi}{3}\right) + i\sin\left(\pi - \frac{\pi}{3}\right)\right]$

$= 2\left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right)$

∴ $\theta = \frac{2\pi}{3}$

10. $|z - zi| = |z||1 - i| = 1$

∴ $|z| = \frac{1}{\sqrt{2}}$

∴ $x^2 + y^2 = \frac{1}{2}$

II. Answer the following:

1. Simplify the following and express in the form a + ib.

i.	$i(8 - 3i)$	**[1 Mark]**
ii.	$(2i^3)^2$	**[1 Mark]**
iii.	$(2 + 3i)(1 - 4i)$	**[1 Mark]**
iv.	$\frac{5}{2}i(-4 - 3i)$	**[2 Marks]**
v.	$(1 + 3i)^2(3 + i)$	**[2 Marks]**
vi.	$\frac{4+3i}{1-i}$	**[2 Marks]**
vii.	$\left(1 + \frac{2}{i}\right)\left(3 + \frac{4}{i}\right)(5 + i)^{-1}$	**[3 Marks]**
viii.	$\frac{\sqrt{5}+\sqrt{3}i}{\sqrt{5}-\sqrt{3}i}$	**[3 Marks]**
ix.	$\frac{3i^5 + 2i^7 + i^9}{i^6 + 2i^8 + 3i^{18}}$	**[3 Marks]**
x.	$\frac{5+7i}{4+3i} + \frac{5+7i}{4-3i}$	**[3 Marks]**

Solution:

i. $3 + \sqrt{-64} = 3 + \sqrt{64}\,.\,\sqrt{-1} = 3 + 8i$

ii. $(2i^3)^2 = 4i^6 = 4(i^2)^3$

$= 4(-1)^3$ …[∵ $i^2 = -1$]

$= -4 = -4 + 0i$

iii. $(2 + 3i)(1 - 4i) = 2 - 8i + 3i - 12i^2$

$= 2 - 5i - 12(-1)$ …[∵ $i^2 = -1$]

$= 14 - 5i$

iv. $\frac{5}{2}i(-4 - 3i) = \frac{5}{2}(-4i - 3i^2)$

$= \frac{5}{2}[-4i - 3(-1)]$ …[∵ $i^2 = -1$]

$= \frac{5}{2}(3 - 4i) = \frac{15}{2} - 10i$

v. $(1 + 3i)^2(3 + i) = (1 + 6i + 9i^2)(3 + i)$

$= (1 + 6i - 9)(3 + i)$ …[∵ $i^2 = -1$]

$= (-8 + 6i)(3 + i)$

$= -24 - 8i + 18i + 6i^2$

$= -24 + 10i + 6(-1)$

$= -24 + 10i - 6 = -30 + 10i$

vi. $\frac{4+3i}{1-i} = \frac{(4+3i)(1+i)}{(1-i)(1+i)} = \frac{4+4i+3i+3i^2}{1-i^2}$

$= \frac{4+7i+3(-1)}{1-(-1)}$ ...[$\because$ $i^2 = -1$]

$= \frac{1+7i}{2} = \frac{1}{2} + \frac{7}{2}i$

---

vii. $\left(1+\frac{2}{i}\right)\left(3+\frac{4}{i}\right)(5+i)^{-1} = \frac{(i+2)}{i}.\frac{(3i+4)}{i}.\frac{1}{5+i}$

$= \frac{3i^2+4i+6i+8}{i^2(5+i)} = \frac{-3+10i+8}{-1(5+i)}$ ...[$\because$ $i^2 = -1$]

$= \frac{(5+10i)}{-(5+i)} = \frac{(5+10i)(5-i)}{-(5+i)(5-i)}$

$= \frac{25-5i+50i-10i^2}{-(25-i^2)}$

$= \frac{25+45i-10(-1)}{-[25-(-1)]} = \frac{35+45i}{-26} = \frac{-35}{26} - \frac{45}{26}i$

---

viii. $\frac{\sqrt{5}+\sqrt{3}i}{\sqrt{5}-\sqrt{3}i} = \frac{(\sqrt{5}+\sqrt{3}i)(\sqrt{5}+\sqrt{3}i)}{(\sqrt{5}-\sqrt{3}i)(\sqrt{5}+\sqrt{3}i)}$

$= \frac{5+2\sqrt{15}i+3i^2}{5-3i^2}$

$= \frac{5+2\sqrt{15}i+3(-1)}{5-3(-1)}$ ...[$\because$ $i^2 = -1$]

$= \frac{2+2\sqrt{15}i}{8} = \frac{1+\sqrt{15}i}{4} = \frac{1}{4} + \frac{\sqrt{15}i}{4}$

---

ix. $\frac{3i^5+2i^7+i^9}{i^6+2i^8+3i^{18}} = \frac{3(i^4.i)+2(i^4.i^3)+(i^4)^2.i}{i^4.i^2+2(i^4)^2+3(i^2)^9}$

$= \frac{3(1).i+2(1)(-i)+(1)^2.i}{(1)(-1)+2(1)^2+3(-1)^9}$

...[$\because$ $i^2 = -1$, $i^3 = -i$, $i^4 = 1$]

$= \frac{3i-2i+i}{-1+2-3} = \frac{2i}{-2} = -i$

---

x. $\frac{5+7i}{4+3i} + \frac{5+7i}{4-3i} = (5+7i)\left[\frac{1}{4+3i} + \frac{1}{4-3i}\right]$

$= (5+7i)\left[\frac{4-3i+4+3i}{(4+3i)(4-3i)}\right]$

$= (5+7i)\left[\frac{8}{16-9i^2}\right]$

$= (5+7i).\left[\frac{8}{16-9(-1)}\right]$ ...[$\because$ $i^2 = -1$]

$= \frac{8(5+7i)}{25} = \frac{40+56i}{25} = \frac{40}{25} + \frac{56}{25}i$

2. Solve the following equations for $x, y \in$ R

i. $(4-5i)x + (2+3i)y = 10-7i$ **[2 Marks]**

ii. $\frac{x+iy}{2+3i} = 7-i$ **[2 Marks]**

iii. $(x+iy)(5+6i) = 2+3i$ **[3 Marks]**

iv. $2x + i^9 y(2+i) = x i^7 + 10 i^{16}$ **[3 Marks]**

Solution:

i. $(4-5i)x + (2+3i)y = 10-7i$

$\therefore$ $(4x+2y) + (3y-5x)i = 10-7i$

Equating real and imaginary parts, we get

$4x + 2y = 10$

i.e., $2x + y = 5$ ...(i)

and $3y - 5x = -7$ ...(ii)

Equation (i) × 3 – equation (ii) gives

$11x = 22$

$\therefore$ $x = 2$

Putting $x = 2$ in (i), we get

$2(2) + y = 5$

$\therefore$ $y = 1$

$\therefore$ $x = 2$ and $y = 1$

---

ii. $\frac{x+iy}{2+3i} = 7-i$

$\therefore$ $x + iy = (7-i)(2+3i)$

$\therefore$ $x + iy = 14 + 21i - 2i - 3i^2 = 14 + 19i - 3(-1)$

$\therefore$ $x + iy = 17 + 19i$

Equating real and imaginary parts, we get

$x = 17$ and $y = 19$

---

iii. $(x+iy)(5+6i) = 2+3i$

$\therefore$ $x + iy = \frac{2+3i}{5+6i}$

$\therefore$ $x + iy = \frac{(2+3i)(5-6i)}{(5+6i)(5-6i)}$

$= \frac{10-12i+15i-18i^2}{25-36i^2} = \frac{10+3i-18(-1)}{25-36(-1)}$

$\therefore$ $x + iy = \frac{28+3i}{61} = \frac{28}{61} + \frac{3}{61}i$

Equating real and imaginary parts, we get

$x = \frac{28}{61}$ and $y = \frac{3}{61}$

---

iv. $2x + i^9 y(2+i) = x i^7 + 10 i^{16}$

$\therefore$ $2x + (i^4)^2.i.y(2+i) = x(i^2)^3.i + 10.(i^4)^4$

$\therefore$ $2x + (1)^2.iy(2+i) = x(-1)^3.i + 10(1)^4$

...[$\because$ $i^2 = -1$, $i^4 = 1$]

$\therefore$ $2x + 2yi + yi^2 = -xi + 10$

$\therefore$ $2x + 2yi - y + xi = 10$

$\therefore$ $(2x - y) + (x + 2y)i = 10 + 0.i$

Equating real and imaginary parts, we get

$2x - y = 10$ ...(i)

and $x + 2y = 0$ ...(ii)

Equation (i) × 2 + equation (ii) gives, we get

$5x = 20$

$\therefore$ $x = 4$

Putting $x = 4$ in (i), we get

$2(4) - y = 10$

$\therefore$ $y = 8 - 10$

$\therefore$ $y = -2$

$\therefore$ $x = 4$ and $y = -2$

---

3. Evaluate

i. $\mathbf{(1 - i + i^2)^{-15}}$ **[2 Marks]**

ii. $\mathbf{i^{131} + i^{49}}$ **[2 Marks]**

Solution:

i. $(1 - i + i^2)^{-15} = (1 - i - 1)^{-15}$

$= (-i)^{-15}$

$= \dfrac{1}{(-i)^{15}}$

$= \dfrac{-1}{(i^4)^3 . i^3}$

$= \dfrac{-1}{(1)^3(-i)}$

$= \dfrac{1}{i} = \dfrac{i}{i^2} = \dfrac{i}{-1} = -i$

---

ii. $i^{131} + i^{49} = (i^4)^{32}\, i^3 + (i^4)^{12} i$

$= (1)^{32}(-i) + (1)^{12} i$

$= -i + i$

$= 0$

---

4. Find the value of

i. $\mathbf{x^3 + 2x^2 - 3x + 21}$**, if** $\mathbf{x = 1 + 2i}$ **[3 Marks]**

ii. $\mathbf{x^4 + 9x^3 + 35x^2 - x + 164}$**, if** $\mathbf{x = -5 + 4i}$ **[4 Marks]**

Solution:

i. $x = 1 + 2i$

$\therefore$ $x - 1 = 2i$

$\therefore$ $(x - 1)^2 = 4i^2$

$\therefore$ $x^2 - 2x + 1 = -4$ …[$\because$ $i^2 = -1$]

$\therefore$ $x^2 - 2x + 5 = 0$ …(i)

$\because$

$$\begin{array}{r|l} & x + 4 \\ \hline x^2 - 2x + 5 & x^3 + 2x^2 - 3x + 21 \\ & x^3 - 2x^2 + 5x \\ & -\quad +\quad - \\ \hline & 4x^2 - 8x + 21 \\ & 4x^2 - 8x + 20 \\ & -\quad +\quad - \\ \hline & 1 \end{array}$$

$\therefore$ $x^3 + 2x^2 - 3x + 21$

$= (x^2 - 2x + 5)(x + 4) + 1$

$= 0.(x + 4) + 1$ …[From (i)]

$= 0 + 1$

$\therefore$ $x^3 + 2x^2 - 3x + 21 = 1$

ii. $x = -5 + 4i$

$\therefore$ $x + 5 = 4i$

$\therefore$ $(x + 5)^2 = 16i^2$

$\therefore$ $x^2 + 10x + 25 = -16$ …[$\because$ $i^2 = -1$]

$\therefore$ $x^2 + 10x + 41 = 0$ …(i)

$$\begin{array}{r|l} & x^2 - x + 4 \\ \hline x^2 + 10x + 41 & x^4 + 9x^3 + 35x^2 - x + 164 \\ & x^4 + 10x^3 + 41x^2 \\ & -\quad -\quad - \\ \hline & -x^3 - 6x^2 - x + 164 \\ & -x^3 - 10x^2 - 41x \\ & +\quad +\quad + \\ \hline & 4x^2 + 40x + 164 \\ & 4x^2 + 40x + 164 \\ & -\quad -\quad - \\ \hline & 0 \end{array}$$

$\therefore$ $x^4 + 9x^3 + 35x^2 - x + 164$

$= (x^2 + 10x + 41)(x^2 - x + 4)$

$= 0(x^2 - x + 4)$ …[From (i)]

$\therefore$ $x^4 + 9x^3 + 35x^2 - x + 164 = 0$

---

5. Find the square roots of **[3 Marks Each]**

i. $\mathbf{-16 + 30i}$ **ii.** $\mathbf{15 - 8i}$

iii. $\mathbf{2 + 2\sqrt{3}\, i}$ **iv.** $\mathbf{18i}$

v. $\mathbf{3 - 4i}$ **vi.** $\mathbf{6 + 8i}$

Solution:

i. Let $\sqrt{-16 + 30i} = a + bi$, where $a, b \in R$.

Squaring on both sides, we get

$-16 + 30i = a^2 + b^2 i^2 + 2abi$

$\therefore$ $-16 + 30i = (a^2 - b^2) + 2abi$ …[$\because$ $i^2 = -1$]

Equating real and imaginary parts, we get

$a^2 - b^2 = -16$ and $2ab = 30$

$\therefore$ $a^2 - b^2 = -16$ and $b = \dfrac{15}{a}$

$\therefore$ $a^2 - \left(\dfrac{15}{a}\right)^2 = -16$

$\therefore$ $a^2 - \dfrac{225}{a^2} = -16$

$\therefore$ $a^4 - 225 = -16a^2$

$\therefore$ $a^4 + 16a^2 - 225 = 0$

$\therefore$ $(a^2 + 25)(a^2 - 9) = 0$

$\therefore$ $a^2 = -25$ or $a^2 = 9$

But $a \in R$

$\therefore$ $a^2 \neq -25$

$\therefore$ $a^2 = 9$

$\therefore$ $a = \pm 3$

When $a = 3$, $b = \dfrac{15}{3} = 5$

When $a = -3$, $b = \dfrac{15}{-3} = -5$

$\therefore$ $\sqrt{-16 + 30i} = \pm(3 + 5i)$

ii. Let $\sqrt{15-8i} = a + bi$, where a, b $\in$ R.

Squaring on both sides, we get

$15 - 8i = a^2 + b^2i^2 + 2abi$

$\therefore$ $15 - 8i = (a^2 - b^2) + 2abi$ …[$\because$ $i^2 = -1$]

Equating real and imaginary parts, we get

$a^2 - b^2 = 15$ and $2ab = -8$

$\therefore$ $a^2 - b^2 = 15$ and $b = \frac{-4}{a}$

$\therefore$ $a^2 - \left(\frac{-4}{a}\right)^2 = 15$

$\therefore$ $a^2 - \frac{16}{a^2} = 15$

$\therefore$ $a^4 - 16 = 15a^2$

$\therefore$ $a^4 - 15a^2 - 16 = 0$

$\therefore$ $(a^2 - 16)(a^2 + 1) = 0$

$\therefore$ $a^2 = 16$ or $a^2 = -1$

But a $\in$ R

$\therefore$ $a^2 \neq -1$

$\therefore$ $a^2 = 16$

$\therefore$ $a = \pm 4$

When $a = 4$, $b = \frac{-4}{4} = -1$

When $a = -4$, $b = \frac{-4}{-4} = 1$

$\therefore$ $\sqrt{15-8i} = \pm (4 - i)$

---

iii. Let $\sqrt{2+2\sqrt{3}i} = a + bi$, where a, b $\in$ R.

Squaring on both sides, we get

$2 + 2\sqrt{3}\,i = a^2 + b^2i^2 + 2abi$

$\therefore$ $2 + 2\sqrt{3}\,i = a^2 - b^2 + 2abi$ …[$\because$ $i^2 = -1$]

Equating real and imaginary parts, we get

$a^2 - b^2 = 2$ and $2ab = 2\sqrt{3}$

$\therefore$ $a^2 - b^2 = 2$ and $b = \frac{\sqrt{3}}{a}$

$\therefore$ $a^2 - \left(\frac{\sqrt{3}}{a}\right)^2 = 2$

$\therefore$ $a^2 - \frac{3}{a^2} = 2$

$\therefore$ $a^4 - 3 = 2a^2$

$\therefore$ $a^4 - 2a^2 - 3 = 0$

$\therefore$ $(a^2 - 3)(a^2 + 1) = 0$

$\therefore$ $a^2 = 3$ or $a^2 = -1$

But a $\in$ R,

$\therefore$ $a^2 \neq -1$

$\therefore$ $a^2 = 3$

$\therefore$ $a = \pm\sqrt{3}$

When $a = \sqrt{3}$, $b = \frac{\sqrt{3}}{\sqrt{3}} = 1$

When $a = -\sqrt{3}$, $b = \frac{\sqrt{3}}{-\sqrt{3}} = -1$

$\therefore$ $\sqrt{2+2\sqrt{3}i} = \pm\left(\sqrt{3} + i\right)$

---

iv. Let $\sqrt{18i} = a + bi$, where a, b $\in$ R.

Squaring on both sides, we get

$18i = a^2 + b^2i^2 + 2abi$

$\therefore$ $0 + 18i = a^2 - b^2 + 2abi$ …[$\because$ $i^2 = -1$]

Equating real and imaginary parts, we get

$a^2 - b^2 = 0$ and $2ab = 18$

$\therefore$ $a^2 - b^2 = 0$ and $b = \frac{9}{a}$

$\therefore$ $a^2 - \left(\frac{9}{a}\right)^2 = 0$

$\therefore$ $a^2 - \frac{81}{a^2} = 0$

$\therefore$ $a^4 - 81 = 0$

$\therefore$ $(a^2 - 9)(a^2 + 9) = 0$

$\therefore$ $a^2 = 9$ or $a^2 = -9$

But a $\in$ R

$\therefore$ $a^2 \neq -9$

$\therefore$ $a^2 = 9$

$\therefore$ $a = \pm 3$

When $a = 3$, $b = \frac{9}{3} = 3$

When $a = -3$, $b = \frac{9}{-3} = -3$

$\therefore$ $\sqrt{18i} = \pm (3 + 3i) = \pm 3(1 + i)$

---

v. Let $\sqrt{3-4i} = a + bi$, where a, b $\in$ R.

Squaring on both sides, we get

$3 - 4i = a^2 + b^2i^2 + 2abi$

$\therefore$ $3 - 4i = a^2 - b^2 + 2abi$ …[$\because$ $i^2 = -1$]

Equating real and imaginary parts, we get

$a^2 - b^2 = 3$ and $2ab = -4$

$\therefore$ $a^2 - b^2 = 3$ and $b = \frac{-2}{a}$

$\therefore$ $a^2 - \left(-\frac{2}{a}\right)^2 = 3$

$\therefore$ $a^2 - \frac{4}{a^2} = 3$

$\therefore$ $a^4 - 4 = 3a^2$

$\therefore$ $a^4 - 3a^2 - 4 = 0$

$\therefore$ $(a^2 - 4)(a^2 + 1) = 0$

$\therefore$ $a^2 = 4$ or $a^2 = -1$

But, a $\in$ R

$\therefore$ $a^2 \neq -1$

$\therefore$ $a^2 = 4$

$\therefore$ $a = \pm 2$

When $a = 2$, $b = \frac{-2}{2} = -1$

When $a = -2$, $b = \frac{-2}{-2} = 1$

$\therefore$ $\sqrt{3-4i} = \pm(2-i)$

---

vi. Let $\sqrt{6+8i} = a + bi$, where $a, b \in R$.
Squaring on both sides, we get
$6 + 8i = a^2 + b^2i^2 + 2abi$

$\therefore$ $6 + 8i = a^2 - b^2 + 2abi$ …[$\because$ $i^2 = -1$]

Equating real and imaginary parts, we get
$a^2 - b^2 = 6$ and $2ab = 8$

$\therefore$ $a^2 - b^2 = 6$ and $b = \frac{4}{a}$

$\therefore$ $a^2 - \left(\frac{4}{a}\right)^2 = 6$

$\therefore$ $a^2 - \frac{16}{a^2} = 6$

$\therefore$ $a^4 - 16 = 6a^2$

$\therefore$ $a^4 - 6a^2 - 16 = 0$

$\therefore$ $(a^2 - 8)(a^2 + 2) = 0$

$\therefore$ $a^2 = 8$ or $a^2 = -2$

But $a \in R$

$\therefore$ $a^2 \neq -2$

$\therefore$ $a^2 = 8$

$\therefore$ $a = \pm 2\sqrt{2}$

When $a = 2\sqrt{2}$, $b = \frac{4}{2\sqrt{2}} = \sqrt{2}$

When $a = -2\sqrt{2}$, $b = \frac{4}{-2\sqrt{2}} = -\sqrt{2}$

$\therefore$ $\sqrt{6+8i} = \pm(2\sqrt{2} + \sqrt{2}\,i) = \pm\sqrt{2}\,(2 + i)$

---

6. Find the modulus and argument of each complex number and express it in the polar form. [3 Marks Each]

i. $\mathbf{8 + 15i}$ **ii.** $\mathbf{6 - i}$

iii. $\frac{1+\sqrt{3}i}{2}$ **iv.** $\frac{-1-i}{\sqrt{2}}$

v. $\mathbf{2i}$ **vi.** $\mathbf{-3i}$

vii. $\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}i$

Solution:

i. Let $z = 8 + 15i$

$\therefore$ $a = 8, b = 15, \quad a, b > 0$

$\therefore$ $|z| = r = \sqrt{a^2+b^2} = \sqrt{(8)^2+(15)^2}$
$= \sqrt{64+225} = \sqrt{289} = 17$

Here, (8, 15) lies in 1st quadrant.

$\therefore$ $\theta = \text{amp}(z) = \tan^{-1}\left(\frac{b}{a}\right) = \tan^{-1}\left(\frac{15}{8}\right)$

$\therefore$ The polar form of $z = r(\cos\theta + i\sin\theta)$
$= 17(\cos\theta + i\sin\theta)$, where $\theta = \tan^{-1}\left(\frac{15}{8}\right)$

---

ii. Let $z = 6 - i$

$\therefore$ $a = 6, b = -1, \quad a > 0, b < 0$

$\therefore$ $|z| = r = \sqrt{a^2+b^2} = \sqrt{6^2+(-1)^2} = \sqrt{36+1}$
$= \sqrt{37}$

Here, (6, −1) lies in 4th quadrant.

$\therefore$ $\theta = \text{amp}(z) = \tan^{-1}\left(\frac{b}{a}\right) = \tan^{-1}\left(\frac{-1}{6}\right)$

$\therefore$ The polar form of $z = r(\cos\theta + i\sin\theta)$
$= \sqrt{37}(\cos\theta + i\sin\theta)$

where $\theta = \tan^{-1}\left(-\frac{1}{6}\right)$

---

iii. Let $z = \frac{1+\sqrt{3}i}{2} = \frac{1}{2} + \frac{\sqrt{3}}{2}i$

$\therefore$ $a = \frac{1}{2}, b = \frac{\sqrt{3}}{2}$, $a, b > 0$

$\therefore$ $|z| = r = \sqrt{a^2+b^2}$
$= \sqrt{\left(\frac{1}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2} = \sqrt{\frac{1}{4}+\frac{3}{4}} = 1$

Here, $\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$ lies in 1st quadrant.

$\therefore$ $\theta = \text{amp}(z) = \tan^{-1}\left(\frac{b}{a}\right)$
$= \tan^{-1}\left(\frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}}\right) = \tan^{-1}\left(\sqrt{3}\right) = \frac{\pi}{3}$

$\therefore$ $\theta = 60° = \frac{\pi}{3}$

$\therefore$ The polar form of $z = r(\cos\theta + i\sin\theta)$
$= 1(\cos 60° + i\sin 60°)$
$= 1\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)$

---

iv. Let $z = \frac{-1-i}{\sqrt{2}} = \frac{-1}{\sqrt{2}} - \frac{1}{\sqrt{2}}i$

$\therefore$ $a = \frac{-1}{\sqrt{2}}, b = \frac{-1}{\sqrt{2}}$, $a, b < 0$

$\therefore$ $|z| = r = \sqrt{a^2+b^2} = \sqrt{\left(\frac{-1}{\sqrt{2}}\right)^2+\left(\frac{-1}{\sqrt{2}}\right)^2} = \sqrt{\frac{1}{2}+\frac{1}{2}} = 1$

Here, $\left(\frac{-1}{\sqrt{2}}, \frac{-1}{\sqrt{2}}\right)$ lies in 3rd quadrant.

$\therefore \quad \theta = \text{amp}(z) = \tan^{-1}\left(\frac{b}{a}\right) - \pi$

$$= \tan^{-1}\left(\frac{\frac{-1}{\sqrt{2}}}{\frac{-1}{\sqrt{2}}}\right) - \pi$$

$$= \tan^{-1}(1) - \pi$$

$$= \frac{\pi}{4} - \pi$$

$$= -\frac{3\pi}{4}$$

$\therefore \quad \theta = -\frac{3\pi}{4}$

$\therefore$ The polar form of $z = r(\cos\theta + i\sin\theta)$

$$= 1\left(\cos\frac{-3\pi}{4} + i\sin\frac{-3\pi}{4}\right)$$

---

v. Let $z = 2i = 0 + 2i$

$\therefore \quad a = 0, b = 2$

$\therefore$ z lies on positive imaginary Y-axis.

$\therefore \quad |z| = r = \sqrt{a^2 + b^2} = \sqrt{0^2 + 2^2} = \sqrt{0+4} = 2$

$\therefore \quad \theta = \text{amp}(z) = \frac{\pi}{2}$

$\therefore$ The polar form of $z = r(\cos\theta + i\sin\theta)$

$$= 2\left(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right)$$

---

vi. Let $z = -3i = 0 - 3i$

$\therefore \quad a = 0, b = -3$

z lies on negative imaginary Y-axis.

$\therefore \quad |z| = r = \sqrt{a^2 + b^2} = \sqrt{0^2 + (-3)^2} = \sqrt{0+9} = 3$

$\therefore \quad \theta = \text{amp}(z) = \frac{-\pi}{2}$

$\therefore$ The polar form of $z = r(\cos\theta + i\sin\theta)$

$$= 3\left(\cos\frac{-\pi}{2} + i\sin\frac{-\pi}{2}\right)$$

---

vii. Let $z = \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}i$

$\therefore \quad a = \frac{1}{\sqrt{2}}, b = \frac{1}{\sqrt{2}}, a > 0, b > 0$

$\therefore \quad |z| = r = \sqrt{a^2 + b^2} = \sqrt{\left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{1}{\sqrt{2}}\right)^2}$

$$= \sqrt{\frac{1}{2} + \frac{1}{2}}$$

$$= 1$$

Here, $\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$ lies in 1st quadrant.

$\therefore \quad \theta = \text{amp}(z) = \tan^{-1}\left(\frac{b}{a}\right)$

$$= \tan^{-1}\left(\frac{\frac{1}{\sqrt{2}}}{\frac{1}{\sqrt{2}}}\right)$$

$$= \tan^{-1}(1) = \frac{\pi}{4}$$

$\therefore \quad \theta = \frac{\pi}{4}$

$\therefore$ The polar form of $z = r(\cos\theta + i\sin\theta)$

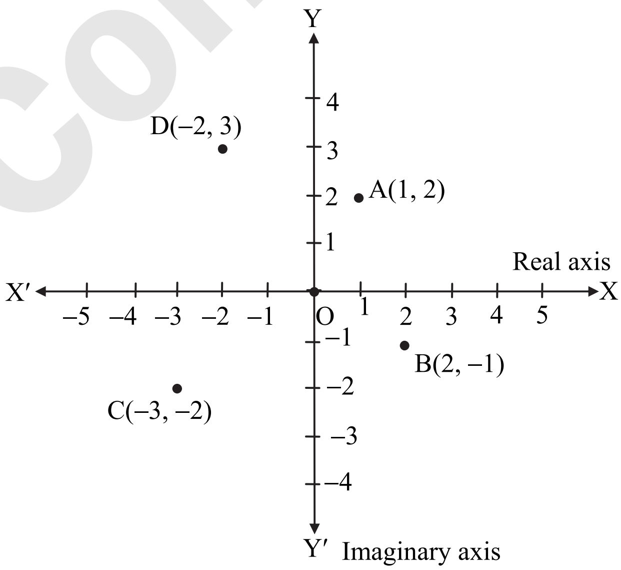
$$= 1\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)$$

---

7. Represent 1 + 2i, 2 – i, –3 – 2i, – 2 + 3i by points in Argand's diagram. [4 Marks]

Solution:

The complex numbers $1+2i, 2-i, -3-2i, -2+3i$ will be represented by the points A(1, 2), B(2, –1), C(–3, –2), D(–2, 3) respectively as shown below:



---

8. Show that $z = \frac{5}{(1-i)(2-i)(3-i)}$ is purely imaginary number. [3 Marks]

Solution:

$z = \frac{5}{(1-i)(2-i)(3-i)}$

$$= \frac{5}{(2-i-2i+i^2)(3-i)}$$

$$= \frac{5}{(2-3i-1)(3-i)} \qquad \ldots[\because i^2 = -1]$$

$$= \frac{5}{(1-3i)(3-i)}$$

$$= \frac{5}{3-i-9i+3i^2}$$

$= \frac{5}{3-10i-3} = \frac{5}{-10i}$

$= \frac{5i}{-10i^2}$

$= \frac{5i}{10}$

$= \frac{1}{2}$ i, which is a purely imaginary number.

---

9. Find the real numbers x and y such that $\frac{x}{1+2i} + \frac{y}{3+2i} = \frac{5+6i}{-1+8i}$. **[3 Marks]**

Solution:

$\frac{x}{1+2i} + \frac{y}{3+2i} = \frac{5+6i}{-1+8i}$

$\therefore \quad \frac{x(3+2i)+y(1+2i)}{(3+2i)(1+2i)} = \frac{5+6i}{-1+8i}$

$\therefore \quad \frac{3x+2xi+y+2yi}{3+6i+2i+4i^2} = \frac{5+6i}{-1+8i}$

$\therefore \quad \frac{(3x+y)+(2x+2y)i}{3+8i+4(-1)} = \frac{5+6i}{-1+8i}$ …[$i^2 = -1$]

$\therefore \quad \frac{(3x+y)+(2x+2y)i}{-1+8i} = \frac{5+6i}{-1+8i}$

$\therefore \quad (3x + y) + 2(x + y)i = 5 + 6i$

Equating real and imaginary parts, we get

$3x + y = 5$ …(i)

and $2(x + y) = 6$

i.e., $x + y = 3$ …(ii)

Subtracting (ii) from (i), we get

$2x = 2$

$\therefore \quad x = 1$

Putting $x = 1$ in (ii), we get

$1 + y = 3$

$\therefore \quad y = 2$

$\therefore \quad x = 1, y = 2$

---

10. Show that $\left(\frac{1}{\sqrt{2}}+\frac{i}{\sqrt{2}}\right)^{10} + \left(\frac{1}{\sqrt{2}}-\frac{i}{\sqrt{2}}\right)^{10} = 0$. **[3 Marks]**

Solution:

$\left(\frac{1}{\sqrt{2}}+\frac{i}{\sqrt{2}}\right)^2 = \frac{1}{2} + 2\left(\frac{1}{\sqrt{2}}\right)\left(\frac{i}{\sqrt{2}}\right) + \frac{i^2}{2}$

$= \frac{1}{2} + i - \frac{1}{2} = i$

$\therefore \quad \left(\frac{1}{\sqrt{2}}+\frac{i}{\sqrt{2}}\right)^{10} = \left[\left(\frac{1}{\sqrt{2}}+\frac{i}{\sqrt{2}}\right)^2\right]^5$

$= i^5 = i^4.i = i$ …(i)

Also, $\left(\frac{1}{\sqrt{2}}-\frac{i}{\sqrt{2}}\right)^2 = \frac{1}{2} - 2\left(\frac{1}{\sqrt{2}}\right)\left(\frac{i}{\sqrt{2}}\right) + \frac{i^2}{2}$

$= \frac{1}{2} - i - \frac{1}{2} = -i$

$\therefore \quad \left(\frac{1}{\sqrt{2}}-\frac{i}{\sqrt{2}}\right)^{10} = \left[\left(\frac{1}{\sqrt{2}}-\frac{i}{\sqrt{2}}\right)^2\right]^5 = (-i)^5$

$= i^4(-i) = -i$ ...(ii)

Adding (i) and (ii), we get

$\left(\frac{1}{\sqrt{2}}+\frac{i}{\sqrt{2}}\right)^{10} + \left(\frac{1}{\sqrt{2}}-\frac{i}{\sqrt{2}}\right)^{10} = i - i = 0$

---

11. Show that $\left(\frac{1+i}{\sqrt{2}}\right)^8 + \left(\frac{1-i}{\sqrt{2}}\right)^8 = 2$. **[3 Marks]**

Solution:

$\left(\frac{1+i}{\sqrt{2}}\right)^2 = \frac{1+2i+i^2}{2}$

$= \frac{1+2i-1}{2} = i$

$\therefore \quad \left(\frac{1+i}{\sqrt{2}}\right)^8 = \left[\left(\frac{1+i}{\sqrt{2}}\right)^2\right]^4 = i^4 = 1$ …(i)

Also, $\left(\frac{1-i}{\sqrt{2}}\right)^2 = \frac{1-2i+i^2}{2} = \frac{1-2i-1}{2} = -i$

$\therefore \quad \left(\frac{1-i}{\sqrt{2}}\right)^8 = \left[\left(\frac{1-i}{\sqrt{2}}\right)^2\right]^4$

$= (-i)^4 = (-1)^4 \times (i)^4$

$= 1 \times i^4 = 1$ …(ii)

Adding (i) and (ii), we get

$\left(\frac{1+i}{\sqrt{2}}\right)^8 + \left(\frac{1-i}{\sqrt{2}}\right)^8 = 1 + 1 = 2$

---

12. Convert the complex numbers in polar form and also in exponential form.

i. $z = \frac{2+6\sqrt{3}i}{5+\sqrt{3}i}$ **[4 Marks]**

ii. $z = -6 + \sqrt{2}\,i$ **[3 Marks]**

iii. $\frac{-3}{2} + \frac{3\sqrt{3}i}{2}$ **[4 Marks]**

Solution:

i. $z = \frac{2+6\sqrt{3}i}{5+\sqrt{3}i}$

$= \frac{(2+6\sqrt{3}i)(5-\sqrt{3}i)}{(5+\sqrt{3}i)(5-\sqrt{3}i)}$

$= \frac{10-2\sqrt{3}i+30\sqrt{3}i-6(3)i^2}{25-3i^2}$

$= \frac{10+28\sqrt{3}i+18}{25+3}$ …[$\because i^2 = -1$]

$= \frac{28+28\sqrt{3}i}{28} = 1 + \sqrt{3}\,i$

$\therefore \quad a = 1, b = \sqrt{3}$, i.e. a, b > 0

$\therefore \quad r = \sqrt{a^2+b^2} = \sqrt{1^2+(\sqrt{3})^2} = \sqrt{1+3} = 2$

Here, $(1, \sqrt{3})$ lies in 1st quadrant.

$\therefore$ $\theta = \text{amp}(z) = \tan^{-1}\left(\frac{b}{a}\right)$

$= \tan^{-1}\left(\frac{\sqrt{3}}{1}\right)$

$= \frac{\pi}{3}$

$\therefore$ $\theta = 60° = \frac{\pi}{3}$

$\therefore$ The polar form of $z = r(\cos\theta + i\sin\theta)$

$= 2(\cos 60° + i \sin 60°)$

$= 2\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)$

$\therefore$ The exponential form of $z = re^{i\theta} = 2e^{\frac{\pi}{3}i}$

---

ii. $z = -6 + \sqrt{2}\,i$

$\therefore$ $a = -6, b = \sqrt{2}$, i.e. $a < 0, b > 0$

$\therefore$ $r = \sqrt{a^2+b^2} = \sqrt{(-6)^2 + (\sqrt{2})^2} = \sqrt{36+2} = \sqrt{38}$

Here, $(-6, \sqrt{2})$ lies in 2nd quadrant.

$\therefore$ $\theta = \text{amp}(z) = \pi + \tan^{-1}\left(\frac{b}{a}\right) = \pi + \tan^{-1}\left(\frac{\sqrt{2}}{-6}\right)$

$= \pi - \tan^{-1}\left(\frac{\sqrt{2}}{6}\right)$

$\therefore$ $\tan^{-1}\left(\frac{\sqrt{2}}{6}\right) = \pi - \theta$

$\therefore$ $\frac{\sqrt{2}}{6} = \tan(\pi - \theta) = -\tan\theta$

$\therefore$ $\theta = \tan^{-1}\left(\frac{-\sqrt{2}}{6}\right)$

$\therefore$ The polar form of $z = r(\cos\theta + i\sin\theta)$

$= \sqrt{38}(\cos\theta + i\sin\theta)$,

where $\theta = \tan^{-1}\left(\frac{-\sqrt{2}}{6}\right)$

$\therefore$ The exponential form of $z = re^{i\theta} = \sqrt{38}\,e^{i\theta}$

---

iii. Let $z = \frac{-3}{2} + \frac{3\sqrt{3}}{2}i$

$\therefore$ $a = \frac{-3}{2}, b = \frac{3\sqrt{3}}{2}$, $a < 0, b > 0$

$\therefore$ $r = \sqrt{a^2+b^2}$

$= \sqrt{\left(\frac{-3}{2}\right)^2 + \left(\frac{3\sqrt{3}}{2}\right)^2} = \sqrt{\frac{9}{4}+\frac{27}{4}} = 3$

Here, $\left(\frac{-3}{2}, \frac{3\sqrt{3}}{2}\right)$ lies in 2nd quadrant.

$\therefore$ $\theta = \text{amp}(z) = \tan^{-1}\left(\frac{b}{a}\right) + \pi$

$= \tan^{-1}\left(\frac{\frac{3\sqrt{3}}{2}}{\frac{-3}{2}}\right) + \pi$

$= \tan^{-1}(-\sqrt{3}) + \pi$

$= \pi - \frac{\pi}{3} = \frac{2\pi}{3}$

$\therefore$ $\theta = \frac{2\pi}{3}$

$\therefore$ The polar form of $z = r(\cos\theta + i.\sin\theta)$

$= 3\left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right)$

$\therefore$ The exponential form of $z = re^{i\theta} = 3e^{\frac{2\pi}{3}i}$

---

13. If $x + iy = \frac{a+ib}{a-ib}$, prove that $x^2 + y^2 = 1$. **[3 Marks]**

Solution:

$x + iy = \frac{a+ib}{a-ib} = \frac{(a+ib)(a+ib)}{(a-ib)(a+ib)}$

$= \frac{a^2 + i^2b^2 + 2abi}{a^2 - i^2b^2}$

$= \frac{(a^2-b^2) + 2abi}{a^2+b^2}$ ...[$\because i^2 = -1$]

$\therefore$ $x + iy = \frac{a^2-b^2}{a^2+b^2} + \frac{2ab}{a^2+b^2}i$

Equating real and imaginary parts, we get

$x = \frac{a^2-b^2}{a^2+b^2}$ and $y = \frac{2ab}{a^2+b^2}$

$\therefore$ $x^2 + y^2 = \frac{(a^2-b^2)^2}{(a^2+b^2)^2} + \frac{4a^2b^2}{(a^2+b^2)^2}$

$= \frac{a^4 + b^4 - 2a^2b^2 + 4a^2b^2}{(a^2+b^2)^2}$

$= \frac{(a^2+b^2)^2}{(a^2+b^2)^2}$

$\therefore$ $x^2 + y^2 = 1$

---

14. Show that $z = \left(\frac{-1+i\sqrt{3}}{2}\right)^3$ is a rational number. **[3 Marks]**

Solution:

$\left(\frac{-1+\sqrt{-3}}{2}\right)^3 = \frac{(-1+\sqrt{3}.\sqrt{-1})^3}{8}$

$= \frac{(-1+\sqrt{3}i)^3}{8}$

$$= \frac{(-1)^3 + 3(-1)^2(i\sqrt{3}) + 3(-1)(i\sqrt{3})^2 + (i\sqrt{3})^3}{8}$$

$$= \frac{-1 + 3\sqrt{3}i - 3\times 3(-1) - 3\sqrt{3}i}{8}$$

$$\ldots[\because i^2 = -1,\ i^3 = -i]$$

$$= \frac{-1+9}{8} = \frac{8}{8}$$

= 1, which is a rational number.

---

15. Show that $\frac{1-2i}{3-4i} + \frac{1+2i}{3+4i}$ is real. [2 Marks]

Solution:

$$\frac{1-2i}{3-4i} + \frac{1+2i}{3+4i} = \frac{(1-2i)(3+4i) + (3-4i)(1+2i)}{(3+4i)(3-4i)}$$

$$= \frac{3 + 4i - 6i - 8i^2 + 3 + 6i - 4i - 8i^2}{9 - 16i^2}$$

$$= \frac{6 - 16i^2}{9 - 16(-1)}$$

$$= \frac{6 - 16(-1)}{9 + 16} \qquad \ldots[\because i^2 = -1]$$

$= \frac{22}{25}$, which is a real number.

---

16. Simplify [3 Marks Each]

i. $\frac{i^{29} + i^{39} + i^{49}}{i^{30} + i^{40} + i^{50}}$ **ii.** $\left(i^{65} + \frac{1}{i^{145}}\right)$

iii. $\frac{i^{238} + i^{236} + i^{234} + i^{232} + i^{230}}{i^{228} + i^{226} + i^{224} + i^{222} + i^{220}}$

Solution:

i. $$\frac{i^{29} + i^{39} + i^{49}}{i^{30} + i^{40} + i^{50}} = \frac{i^{29} + i^{39} + i^{49}}{i\left(i^{29} + i^{39} + i^{49}\right)} = \frac{1}{i} = \frac{i}{i^2} = \frac{i}{-1} = -i$$

ii. $$\left(i^{65} + \frac{1}{i^{145}}\right) = \left[(i^4)^{16}.i + \frac{1}{(i^4)^{36}.i}\right] = i + \frac{1}{i}$$

$$= \frac{i^2 + 1}{i} = \frac{-1+1}{i} = 0$$

iii. $$\frac{i^{238} + i^{236} + i^{234} + i^{232} + i^{230}}{i^{228} + i^{226} + i^{224} + i^{222} + i^{220}}$$

$$= \frac{i^{10}(i^{228} + i^{226} + i^{224} + i^{222} + i^{220})}{i^{228} + i^{226} + i^{224} + i^{222} + i^{220}}$$

$= i^{10} = (i^4)^2.\ i^2 = (1)^2\ (-1)$

$= -1$

---

17. Simplify $\left[\frac{1}{1-2i} + \frac{3}{1+i}\right]\left[\frac{3+4i}{2-4i}\right]$ [3 Marks]

Solution:

$$\left[\frac{1}{1-2i} + \frac{3}{1+i}\right]\left[\frac{3+4i}{2-4i}\right]$$

$$= \left[\frac{1+i+3-6i}{(1-2i)(1+i)}\right]\left[\frac{3+4i}{2-4i}\right]$$

$$= \left[\frac{4-5i}{1+i-2i-2i^2}\right]\left[\frac{3+4i}{2-4i}\right]$$

$$= \frac{(4-5i)(3+4i)}{(3-i)(2-4i)}$$

$$= \frac{12 + 16i - 15i - 20i^2}{6 - 12i - 2i + 4i^2}$$

$$= \frac{12 + i + 20}{6 - 14i - 4} = \frac{32 + i}{2 - 14i}$$

$$= \frac{(32+i)(2+14i)}{(2-14i)(2+14i)} = \frac{64 + 448i + 2i + 14i^2}{4 - 196i^2}$$

$$= \frac{64 + 450i - 14}{4 + 196} = \frac{50 + 450i}{200} = \frac{50}{200}(1 + 9i)$$

$$= \frac{1}{4} + \frac{9}{4}i$$

---

18. If α and β are complex cube roots of unity, prove that $(1-\alpha)(1-\beta)(1-\alpha^2)(1-\beta^2) = 9$. [3 Marks]

Solution:

α and β are the complex cube roots of unity.

$\therefore$ $\alpha = \frac{-1+i\sqrt{3}}{2}$ and $\beta = \frac{-1-i\sqrt{3}}{2}$

$\therefore$ $$\alpha\beta = \left(\frac{-1+i\sqrt{3}}{2}\right)\left(\frac{-1-i\sqrt{3}}{2}\right)$$

$$= \frac{(-1)^2 - (i\sqrt{3})^2}{4}$$

$$= \frac{1-(-1)(3)}{4} = \frac{1+3}{4}$$

$\therefore$ $\alpha\beta = 1$

Also, $$\alpha+\beta = \frac{-1+i\sqrt{3}}{2} + \frac{-1-i\sqrt{3}}{2}$$

$$= \frac{-1+i\sqrt{3}-1-i\sqrt{3}}{2} = \frac{-2}{2}$$

$\therefore$ $\alpha + \beta = -1$

$\therefore$ $(1-\alpha)(1-\beta)(1-\alpha^2)(1-\beta^2)$

$= (1-\alpha)(1-\beta)(1-\alpha)(1+\alpha)(1-\beta)(1+\beta)$

$= (1-\alpha)^2(1-\beta)^2(1+\alpha)(1+\beta)$

$= [(1-\alpha)(1-\beta)]^2\ (1+\alpha)(1+\beta)$

$= (1-\beta-\alpha+\alpha\beta)^2\ (1+\alpha+\beta+\alpha\beta)$

$= [1-(\alpha+\beta)+\alpha\beta]^2\ [1+(\alpha+\beta)+\alpha\beta]$

$= [1-(-1)+1]^2\ (1-1+1)$

$= 3^2\ (1) = 9$

---

19. If ω is a complex cube root of unity, prove that $(1-\omega+\omega^2)^6 + (1+\omega-\omega^2)^6 = 128$. [3 Marks]

Solution:

ω is the complex cube root of unity.

$\therefore$ $\omega^3 = 1$ and $1 + \omega + \omega^2 = 0$

Also, $1 + \omega^2 = -\omega$, $1 + \omega = -\omega^2$

$\therefore$ L.H.S. $= (1 - \omega + \omega^2)^6 + (1 + \omega - \omega^2)^6$
$= [(1 + \omega^2) - \omega]^6 + [(1 + \omega) - \omega^2]^6$
$= (-\omega - \omega)^6 + (-\omega^2 - \omega^2)^6$
$= (-2\omega)^6 + (-2\omega^2)^6$
$= 64\omega^6 + 64\omega^{12}$
$= 64(\omega^3)^2 + 64(\omega^3)^4$
$= 64(1)^2 + 64(1)^4$
$= 128$

20. If ω is the cube root of unity, then find the value of $\left(\frac{-1+i\sqrt{3}}{2}\right)^{18} + \left(\frac{-1-i\sqrt{3}}{2}\right)^{18}$.

[3 Marks]

Solution:

If ω is the complex cube root of unity, then

$\omega^3 = 1$, $\omega = \frac{-1+i\sqrt{3}}{2}$ and $\omega^2 = \left(\frac{-1-i\sqrt{3}}{2}\right)^2$

Consider, $\left(\frac{-1+i\sqrt{3}}{2}\right)^{18} + \left(\frac{-1-i\sqrt{3}}{2}\right)^{18}$

Given Expression $= \omega^{18} + (\omega^2)^{18}$
$= \omega^{18} + \omega^{36}$
$= (\omega^3)^6 + (\omega^3)^{12}$
$= (1)^6 + (1)^{12} = 2$

One Mark Questions

1. Simplify: $\sqrt{289}\,i + 4\sqrt{169}\,i - 3\sqrt{196}\,i$
2. Find the distance of the point P from the origin, where the point P represents the complex number z = 3 + 4i in the plane.
3. If $z = 2 + 2\sqrt{3}\,i$, find the amplitude of z.
4. If ω is a complex cube root of unity, then find the value of ω^{-39}.
5. Express $z = \dfrac{\cos\frac{7\pi}{12} + i\sin\frac{7\pi}{12}}{\sqrt{5}}$ in the exponential form.

Multiple Choice Questions

1. If n is a positive integer, then which of the following relations is false
(A) $i^{4n} = 1$ (B) $i^{4n-1} = i$
(C) $i^{4n+1} = i$ (D) $i^{-4n} = 1$

2. The value of $(1 + i)^5 \times (1 - i)^5$ is
(A) – 8 (B) 8i
(C) 8 (D) 32

3. If $x = 3 + i$, then $x^3 - 3x^2 - 8x + 15 =$
(A) 6 (B) 10
(C) – 18 (D) – 15

4. If $(1 - i)x + (1 + i)y = 1 - 3i$, then $(x, y) =$
(A) (2, –1) (B) (–2, 1)
(C) (–2, –1) (D) (2, 1)

5. $\dfrac{\sqrt{5+12i}+\sqrt{5-12i}}{\sqrt{5+12i}-\sqrt{5-12i}} =$
(A) $-\frac{3}{2}i$ (B) $\frac{3}{2}i$
(C) $-\frac{3}{2}$ (D) $\frac{3}{2}$

6. If $\dfrac{5(-8+6i)}{(1+i)^2} = a + ib$, then (a, b) equals
(A) (15, 20) (B) (20, 15)
(C) (–15, 20) (D) None of these

7. If $\left(\frac{1-i}{1+i}\right)^{100} = a + ib$, then
(A) a = 2, b = –1 (B) a = 1, b = 0
(C) a = 0, b = 1 (D) a = –1, b = 2

8. The conjugate of the complex number $\frac{2+5i}{4-3i}$ is
(A) $\frac{7-26i}{25}$ (B) $\frac{-7-26i}{25}$
(C) $\frac{-7+26i}{25}$ (D) $\frac{7+26i}{25}$

9. $\left|(1+i)\frac{(2+i)}{(3+i)}\right| =$
(A) $-\frac{1}{2}$ (B) $\frac{1}{2}$
(C) 1 (D) –1

10. $\arg\left(\frac{3+i}{2-i} + \frac{3-i}{2+i}\right)$ is equal to
(A) $\frac{\pi}{2}$ (B) $-\frac{\pi}{2}$
(C) 0 (D) $\frac{\pi}{4}$

11. The amplitude of $\frac{1+\sqrt{3}i}{\sqrt{3}-i}$ is
(A) 0 (B) π/6
(C) π/3 (D) π/2

12. The modulus and amplitude of $\frac{1+2i}{1-(1-i)^2}$ are
(A) $\sqrt{2}$ and $\frac{\pi}{6}$ (B) 1 and 0
(C) 1 and $\frac{\pi}{3}$ (D) 1 and $\frac{\pi}{4}$

13. $\sqrt{-8-6i} =$
(A) 1 ± 3i (B) ± (1 – 3i)
(C) ± (1 + 3i) (D) ± (3 – i)

14. The square root of 3 – 4i is
(A) ±(2 + i) (B) ±(2 – i)
(C) ±(1 – 2i) (D) ±(1 + 2i)

15. $(27)^{1/3}$ =
(A) 3 (B) 3, 3i, $3i^2$
(C) 3, 3ω, $3\omega^2$ (D) None of these

16. If ω is a complex cube root of unity, then $(x - y)(x\omega - y)(x\omega^2 - y) =$
(A) $x^2 + y^2$ (B) $x^2 - y^2$
(C) $x^3 - y^3$ (D) $x^3 + y^3$

17. If 1, ω, ω^2 are the three cube roots of unity, then $(3 + \omega^2 + \omega^4)^6 =$
(A) 64 (B) 729
(C) 21 (D) 0

18. If α and β are imaginary cube roots of unity, then the value of $\alpha^4 + \beta^{28} + \frac{1}{\alpha\beta}$ is
(A) 1 (B) –1
(C) 0 (D) None of these

19. If ω is an imaginary cube root of unity, $(1 + \omega - \omega^2)^7$ equals
(A) 128ω (B) –128ω
(C) $128\omega^2$ (D) $-128\omega^2$

Answers

One Mark Questions

1. 27i
2. 5
3. $\frac{\pi}{3}$
4. 1
5. $\frac{1}{\sqrt{5}}e^{i\frac{7\pi}{12}}$

Multiple Choice Questions

1.	(B)	2.	(D)	3.	(D)	4.	(A)
5.	(A)	6.	(A)	7.	(B)	8.	(C)
9.	(C)	10.	(C)	11.	(D)	12.	(B)
13.	(B)	14.	(B)	15.	(C)	16.	(C)
17.	(A)	18.	(C)	19.	(D)		

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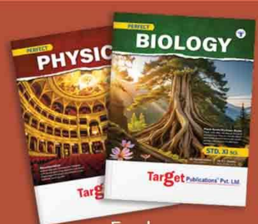

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