SAMPLE CONTENT

Perfect Notes

MATHEMATICS Part - I

STD.IX

(Eng. Med.)

Target Publications Pvt. Ltd.

Written as per the latest syllabus prescribed by the Maharashtra State Bureau of Textbook Production and Curriculum Research, Pune.

Mathematics Part – I

STD. IX

Salient Features

- Written as per the new textbook.
- Exhaustive coverage of entire syllabus.
- Topic-wise distribution of textual questions and practice problems at the beginning of every chapter.
- Covers solutions to all practice sets and problem sets.
- Includes additional problems for practice.
- MCQs for preparation of competitive examinations.
- Includes practice test for each chapter.

Printed at: **India Printing Works,** Mumbai

P.O. No. 124872

TEID: 12561_JUP

PREFACE

Preparing this '**Mathematics Part - I'** book was a rollercoaster ride. We had a plethora of ideas, suggestions and decisions to ponder over. However, our basic premise was to keep this book in line with the new, improved syllabus and to provide students with an absolutely fresh material.

Mathematics Part **-** I covers several topics in the areas of numbers, algebra, commercial mathematics and data handling. The study of these topics requires a deep and intrinsic understanding of concepts, terms and formulae. Hence, to ease this task, we present **'Std. IX: Mathematics Part - I' –** a complete and thorough guide, extensively drafted to boost the confidence of students.

For better understanding of different types of questions, topic-wise distribution of textual questions and practice problems has been provided at the beginning of every chapter. Before each practice set, short and easy explanation of different concepts with illustrations for better understanding is given. Solutions to textual questions and examples are provided in a lucid manner.

'Multiple Choice Questions' based on each chapter facilitate students to prepare for competitive examinations.

'Additional problems for practice' includes additional unsolved problems for practice to help the students sharpen their problem solving skills. 'Solve examples' from textbook are included in this section.

'Apply your knowledge' covers all the textual activities and projects along with their answers.

Every chapter ends with a 'Practice Test'. This test stands as a testimony to the fact that the child has understood the chapter thoroughly.

The journey to create a complete book is strewn with triumphs, failures and near misses. If you think we've nearly missed something or want to applaud us for our triumphs, we'd love to hear from you.

Please write to us on: mail@targetpublications.org

A book affects eternity; one can never tell where its influence stops.

Best of luck to all the aspirants!

From,
Publisher

Edition: First

Contents

Solved examples from textbook are indicated by "+".

2 Real Numbers

Type of Problems	Practice Set	Q. Nos.
Decimal form of rational numbers	2.1	Q.1, 2
	Practice Problems (Based on Practice Set 2.1)	Q.1
	Problem Set-2	Q.3
To express the recurring decimal in $\frac{p}{q}$ form	2.1	Q.3
	Practice Problems (Based on Practice Set 2.1)	Q.2, 3
	Problem Set-2	Q.2
Irrational and real numbers	2.2	Q.1, 2, 3, 4
	Practice Problems (Based on Practice Set 2.2)	Q.1, 2
	Problem Set-2	Q.4
Surds, order of surds	2.3	Q.1, 2
	Practice Problems (Based on Practice Set 2.3)	Q.1
Simplest form of a surd, rationalizing factor	2.3	Q.4
	Practice Problems (Based on Practice Set 2.3)	Q.5, 8
	Problem Set-2	Q. 5, 6
Like surds, unlike surds	2.3	Q.3
Operations on surds	2.3	Q. 6, 7, 8
	Practice Problems (Based on Practice Set 2.3)	Q.3, 6, 7
	2.4	Q.1
	Problem Set-2	Q.7
Comparison of surds	2.3	Q.5
	Practice Problems (Based on Practice Set 2.3)	Q.2
Rationalizing the denominator	2.3	Q.9
	Practice Problems (Based on Practice Set 2.3)	Q.4
	2.4	Q.2
	Practice Problems (Based on Practice Set 2.4)	Q.1
	Problem Set-2	Q.8
Absolute value	2.5	Q.1, 2
	Practice Problems (Based on Practice Set 2.5)	Q.1, 2

Let's Recall

1. Set of numbers:

i. N = Set of Natural numbers
$= \{1, 2, 3, 4, \ldots\}$

ii. W = Set of Whole numbers
$= \{0, 1, 2, 3, 4, \ldots\}$

iii. I = Set of Integers
$= \{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\}$

iv. Q = Set of Rational numbers
$= \left\{\frac{p}{q} \mid p, q, \in I, q \neq 0\right\}$

v. R = Set of Real numbers
$N \subseteq W \subseteq I \subseteq Q \subseteq R$

2. Order relation on rational numbers:

$\frac{p}{q}$ and $\frac{r}{s}$ are any two rational numbers, with $q > 0$, $s > 0$, then

i. If $ps = qr$, then $\frac{p}{q} = \frac{r}{s}$

Example: $\frac{3}{5} = \frac{6}{10}$, because $3 \times 10 = 5 \times 6$

ii. If $ps > qr$, then $\frac{p}{q} > \frac{r}{s}$

Example: $\frac{3}{4} > \frac{2}{5}$, because $3 \times 5 > 2 \times 4$

iii. If $ps < qr$, then $\frac{p}{q} < \frac{r}{s}$

Example: $\frac{1}{5} < \frac{2}{3}$, because $1 \times 3 < 2 \times 5$

Let's Study

Properties of rational numbers

If a, b, c are rational numbers, then

	Property	Addition	Multiplication
1.	Commutative	$a + b = b + a$	$a \times b = b \times a$
2.	Associative	$(a + b) + c = a + (b + c)$	$a \times (b \times c) = (a \times b) \times c$
3.	Identity	$a + 0 = 0 + a = a$	$a \times 1 = 1 \times a = a$
4.	Inverse	$a + (-a) = 0$	$a \times \frac{1}{a} = 1$ $(a \neq 0)$

Let's Recall

Decimal form of Rational numbers:

The decimal form of any rational number is

i. Terminating type or

ii. Non-terminating recurring type

Examples:

Terminating type	Non-terminating recurring type
$\frac{4}{5} = 0.8$	$\frac{1}{3} = 0.333\ldots = 0.\dot{3}$
$\frac{17}{8} = 2.125$	$\frac{2}{7} = 0.285714285714\ldots = 0.\overline{285714}$
$\frac{45}{6} = 7.5$	$\frac{7}{12} = 0.58333\ldots = 0.58\overline{3}$

Remember This

i. If the prime factors of 'q' are any combination of 2 or 5 or both 2 and 5, then the decimal form of the fraction $\frac{p}{q}$ is of terminating type.

ii. If the prime factors are other than 2 or 5, then its decimal expansion is non-terminating and recurring.

Let's Study

To express the recurring decimal in $\frac{p}{q}$ form

Count the number of recurring digits after decimal point in the given rational number, and multiply it by 10, 100, 1000 accordingly.

Examples:

i. In $4.\dot{5}$ digit 5 is the only recurring digit. Hence, to convert $4.\dot{5}$ in $\frac{p}{q}$ form multiply it by 10.

ii. Write $0.\dot{5}$ in $\frac{p}{q}$ form.

Solution:

Let $x = 0.\dot{5}$ …(i)

Multiplying both sides by 10,

$10x = 5.\dot{5}$ …(ii)

Subtracting (i) from (ii),

$10x - x = 5.\dot{5} - 0.\dot{5}$

$\therefore \quad 9x = 5$

$\therefore \quad x = \frac{5}{9}$

$\therefore \quad \mathbf{0.\dot{5} = \frac{5}{9}}$

Try This

1. How to convert $2.4\dot{3}$ in $\frac{p}{q}$ form ?

(Textbook pg. no. 20)

***Solution*:**

Let $x = 2.4\dot{3}$ …(i)

Multiplying both sides by 100,

$100x = 243.3\dot{3}$ …(ii)

Subtracting (i) from (ii),

$100x - x = 243.3\dot{3} - 2.4\dot{3}$

$\therefore \quad 99x = 240.9$

$\therefore \quad x = \frac{240.9}{99}$

$\therefore \quad 2.4\dot{3} = \frac{240.9}{99}$

Hence, to convert $2.4\dot{3}$ in $\frac{p}{q}$ form, we will have to multiply it by 100 and not 10.

Practice Set 2.1

1. Classify the decimal form of the given rational numbers into terminating and non-terminating recurring type.

i. $\frac{13}{5}$ ii. $\frac{2}{11}$

iii. $\frac{29}{16}$ iv. $\frac{17}{125}$

v. $\frac{11}{6}$

***Solution*:**

i. $5 = 1 \times 5 = 2^0 \times 5^1$

Since the denominator is of the form $2^m \times 5^n$, the decimal form of the rational number will be **terminating type**.

ii. $11 = 1 \times 11$

Since the denominator is not of the form $2^m \times 5^n$, the decimal form of the rational number will be **non-terminating recurring type.**

iii. $16 = 2 \times 2 \times 2 \times 2 \times 1 = 2^4 \times 5^0$

Since the denominator is of the form $2^m \times 5^n$, the decimal form of the rational number will be **terminating type**.

iv. $125 = 1 \times 5 \times 5 \times 5 = 2^0 \times 5^3$

Since the denominator is of the form $2^m \times 5^n$, the decimal form of the rational number will be **terminating type**.

v. $6 = 2 \times 3$

Since the denominator is not of the form $2^m \times 5^n$, the decimal form of the rational number will be **non-terminating recurring type.**

2. Write the following rational numbers in decimal form.

i. $\frac{127}{200}$ ii. $\frac{25}{99}$

iii. $\frac{23}{7}$ iv. $\frac{4}{5}$

v. $\frac{17}{8}$

***Solution*:**

i. $\frac{127}{200} = \frac{127}{2 \times 100} = \frac{63.5}{100} = 0.635$

$\therefore \quad \mathbf{\frac{127}{200} = 0.635}$

ii. $\frac{25}{99} = 0.2525.....$

$\therefore \quad \mathbf{\frac{25}{99} = 0.\overline{25}}$

iii. $\frac{23}{7} = 3.285714285714.....$

$\therefore \quad \mathbf{\frac{23}{7} = 3.\overline{285714}}$

iv. $\frac{4}{5} = \frac{4 \times 2}{5 \times 2} = \frac{8}{10} = 0.8$

$\therefore \quad \mathbf{\frac{4}{5} = 0.8}$

v. $\frac{17}{8} = \frac{17 \times 1.25}{8 \times 1.25} = \frac{21.25}{10} = 2.125$

$\therefore \quad \mathbf{\frac{17}{8} = 2.125}$

3. Write the following decimal numbers in $\frac{p}{q}$ form.

i. $0.\dot{6}$ ii. $0.\overline{37}$

iii. $3.\overline{17}$ iv. $15.\overline{89}$

v. $2.\overline{514}$

Solution:

i. Let $x = 0.\dot{6}$ …(i)

Multiplying both sides by 10,

$10x = 6.\dot{6}$ …(ii)

Subtracting (i) from (ii),

$10x - x = 6.\dot{6} - 0.\dot{6}$

$\therefore$ $9x = 6$

$\therefore$ $x = \dfrac{6}{9} = \dfrac{3 \times 2}{3 \times 3}$

$\therefore$ $x = \dfrac{2}{3}$

$\therefore$ $\mathbf{0.\dot{6} = \dfrac{2}{3}}$

---

ii. Let $x = 0.\overline{37}$ …(i)

Multiplying both sides by 100,

$100x = 37.\overline{37}$ …(ii)

Subtracting (i) from (ii),

$100x - x = 37.\overline{37} - 0.\overline{37}$

$\therefore$ $99x = 37$

$\therefore$ $x = \dfrac{37}{99}$

$\therefore$ $\mathbf{0.\overline{37} = \dfrac{37}{99}}$

---

iii. Let $x = 3.\overline{17}$ …(i)

Multiplying both sides by 100,

$100x = 317.\overline{17}$ …(ii)

Subtracting (i) from (ii),

$100x - x = 317.\overline{17} - 3.\overline{17}$

$\therefore$ $99x = 314$

$\therefore$ $x = \dfrac{314}{99}$

$\therefore$ $\mathbf{3.\overline{17} = \dfrac{314}{99}}$

---

iv. Let $x = 15.\overline{89}$ …(i)

Multiplying both sides by 100,

$100x = 1589.\overline{89}$ …(ii)

Subtracting (i) from (ii),

$100x - x = 1589.\overline{89} - 15.\overline{89}$

$\therefore$ $99x = 1574$

$\therefore$ $x = \dfrac{1574}{99}$

$\therefore$ $\mathbf{15.\overline{89} = \dfrac{1574}{99}}$

v. Let $x = 2.\overline{514}$ …(i)

Multiplying both sides by 1000,

$1000x = 2514.\overline{514}$ …(ii)

Subtracting (i) from (ii),

$1000x - x = 2514.\overline{514} - 2.\overline{514}$

$\therefore$ $999x = 2512$

$\therefore$ $x = \dfrac{2512}{999}$

$\therefore$ $\mathbf{2.\overline{514} = \dfrac{2512}{999}}$

Let's Recall

To represent irrational numbers on a number line:

Example: Represent $\sqrt{2}$ on the numberline.

Solution:

In ΔOAB,

m$\angle$OAB = 90°

By Pythagoras theorem,

$(OB)^2 = (OA)^2 + (AB)^2$

$\therefore$ $(OB)^2 = (1)^2 + (1)^2$

$\therefore$ $(OB)^2 = 2$

$\therefore$ OB = $\sqrt{2}$ units

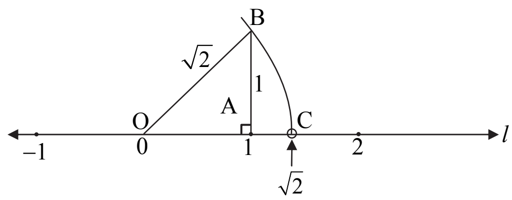
…[Taking square root of both sides]

Steps of construction:

i. Draw a number line and take point A at 1.

ii. Draw AB perpendicular to the number line such that AB = 1 unit.

iii. With O as centre and radius equal to OB, draw an arc to intersect the number line at C.



Let's Study

Irrational and real numbers

1. Irrational Number:

The number which is not rational is called irrational number. The set of irrational numbers is denoted by Q'.

Example: $\sqrt{3}$, $\sqrt{5}$, $2+\sqrt{8}$, π, $-\sqrt{3}$, $-\sqrt{5}$ etc. are irrational numbers.

---

2. Decimal form of irrational numbers:

The decimal form of an irrational number is of non-recurring and non-terminating type.

Examples: i. $\sqrt{2} = 1.41421356\ldots$

ii. $\sqrt{3} = 1.73205080\ldots$

iii. $\sqrt{5} = 2.23606797\ldots$

3. To show that $\sqrt{2}$ is not a rational number:

Let us assume that $\sqrt{2}$ is a rational number.

Then, $\sqrt{2} = \frac{a}{b}$, where 'a' and 'b' have no common factor other than 1 and $b \neq 0$.

$\therefore$ $\sqrt{2} = \frac{a}{b}$, where 'a' and 'b' are co-prime numbers.

$\therefore$ $b\sqrt{2} = a$

$\therefore$ $2b^2 = a^2$ ...(i) [Squaring both sides]

$\therefore$ $b^2 = \frac{a^2}{2}$

Since, 2 divides 'a^2', so 2 divides 'a' as well.
So, we write a = 2c, where c is an integer.

$\therefore$ $a^2 = (2c)^2$ …[Squaring both sides]

$\therefore$ $2b^2 = 4c^2$ …[From (i)]

$\therefore$ $b^2 = 2c^2$

$\therefore$ $c^2 = \frac{b^2}{2}$

Since, 2 divides 'b^2', so 2 divides 'b'.

$\therefore$ 2 divides 'a' and 'b' both.

$\therefore$ 'a' and 'b' have at least 2 as a common factor.

But this contradicts the fact that 'a' and 'b' have no common factor other than 1.

$\therefore$ Our assumption that $\sqrt{2}$ is a rational number is wrong.

$\therefore$ **$\sqrt{2}$ is not a rational number.**

4. Properties of an irrational number:

i. $Q \pm Q' = Q'$

ii. $Q \times Q' = Q'$

iii. $Q \div Q' = Q'$

iv. $Q' \pm Q' = Q$ or Q'

v. $Q' \times Q' = Q$ or Q'

vi. $Q' \div Q' = Q$ or Q'

Here, Q = Non zero rational number

Q′ = Non zero irrational number

Real Number

1. All rational numbers and all irrational numbers together make the set of real numbers.
   $\mathbf{Q \cup Q' = R}$

2. The set of real numbers is denoted by R.

Properties of order relation on Real numbers

1. For any two real numbers a and b, only one of the following relations holds good
   i. $a = b$
   ii. $a < b$
   iii. $a > b$

2. If $a < b$ and $b < c$, then $a < c$

3. If $a < b$, then $a + c < b + c$

4. Let $a < b$,
   i. If $c > 0$, then $ac < bc$
   ii. If $c < 0$, then $ac > bc$

   Examples: a. $2 < 5$, and $3 > 0$
   $\therefore$ $2 \times 3 < 5 \times 3$

   b. $2 < 5$, and $-3 < 0$
   $\therefore$ $2 \times (-3) > 5 \times (-3)$

Square root of Negative number

1. The square root of a negative real number is not a real number.

Remember This

i. Every point on a number line is associated with a unique real number and every real number is associated with a unique point on the number line.

ii. Every rational number is a real number, but every real number may not be a rational number.

iii. The square root of 0 is 0.

Practice Set 2.2

1. Show that $4\sqrt{2}$ is an irrational number.

Proof:

Let us assume that $4\sqrt{2}$ is a rational number.

So, we can find co-prime integers 'a' and 'b' ($b \neq 0$) such that

$4\sqrt{2} = \frac{a}{b}$

$\therefore$ $b\left(4\sqrt{2}\right) = a$

$\therefore$ $32b^2 = a^2$ ... (i) [Squaring both the sides]

$\therefore$ $b^2 = \frac{a^2}{32}$

Since, 32 divides a^2, so 32 divides 'a' as well.
So, we write a = 32c, where c is an integer.

$\therefore$ $a^2 = (32c)^2$ …[Squaring both the sides]

$\therefore$ $32b^2 = 32 \times 32c^2$ …[From (i)]

$\therefore$ $b^2 = 32c^2$

$\therefore$ $c^2 = \dfrac{b^2}{32}$

Since, 32 divides b^2, so 32 divides 'b'.

$\therefore$ 32 divides both a and b.

$\therefore$ a and b have at least 32 as a common factor.

But this contradicts the fact that a and b have no common factor other than 1.

$\therefore$ Our assumption that $4\sqrt{2}$ is a rational number is wrong.

$\therefore$ **$4\sqrt{2}$ is an irrational number.**

Alternate Method:

Let us assume that $4\sqrt{2}$ is a rational number . So, we can find co-prime intergers 'a' and 'b' ($b \neq 0$) such that

$\therefore$ $4\sqrt{2} = \dfrac{a}{b}$

$\therefore$ $\sqrt{2} = \dfrac{a}{4b}$

Since, a and b are integers, $\dfrac{a}{4b}$ is a rational number and so $\sqrt{2}$ is a rational number.

But this contradicts the fact that $\sqrt{2}$ is an irrational number.

$\therefore$ Our assumption is wrong.

$\therefore$ **$4\sqrt{2}$ is an irrational number.**

---

2. Prove that $3+\sqrt{5}$ is an irrational number.

Proof:

Let us assume that $3+\sqrt{5}$ is a rational number. So, we can find co-prime integers 'a' and 'b' ($b \neq 0$) such that

$3+\sqrt{5} = \dfrac{a}{b}$

$\therefore$ $\sqrt{5} = \dfrac{a}{b} - 3$

Since, a and b are integers, $\dfrac{a}{b} - 3$ is a rational number and so $\sqrt{5}$ is a rational number.

But this contradicts the fact that $\sqrt{5}$ is an irrational number.

This contradiction arises because we have

$\therefore$ Our assumption that $3+\sqrt{5}$ is a rational number is wrong.

$\therefore$ **$3+\sqrt{5}$ is an irrational number.**

3. Represent the numbers $\sqrt{5}$ and $\sqrt{10}$ on a number line.

Ans: i. In ΔOAB, m$\angle$OAB = 90°

By Pythagoras theorem,

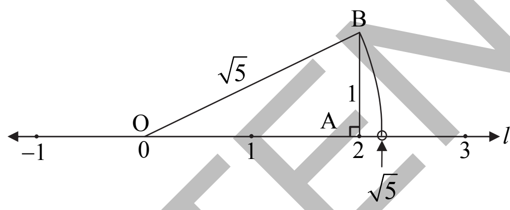
$(OB)^2 = (OA)^2 + (AB)^2$

$= (2)^2 + (1)^2$

$\therefore$ $(OB)^2 = 5$

$\therefore$ $OB = \sqrt{5}$ units.

…[Taking square root of both sides]



ii. In ΔOPR, m$\angle$OPR = 90°

By Pythagoras theorem,

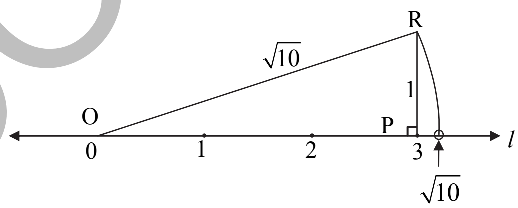
$(OR)^2 = (OP)^2 + (PR)^2$

$= (3)^2 + (1)^2$

$\therefore$ $(OR)^2 = 10$

$\therefore$ $OR = \sqrt{10}$ units.

…[Taking square root of both sides]



---

4. Write any three rational numbers between the two numbers given below.

i. 0.3 and – 0.5

ii. – 2.3 and – 2.33

iii. 5.2 and 5.3

iv. – 4.5 and – 4.6

Ans: i. – 0.4, – 0.3, 0.2

ii. – 2.310, –2.320, –2.325

iii. 5.21, 5.22, 5.23

iv. – 4.51, – 4.55, – 4.58

[**Note:** The above problem has many solutions. Students may write solutions other than the ones given]

Let's Study

Root of positive rational number

1. If n is a positive integer and $x^n = a$, then x is the n^{th} root of a.

This root may be rational or irrational.

Example: $3^4 = 81$

$\therefore$ 3 is the fourth root of 81.

2. If n is an integer greater than 1 and if a is a positive real number, and n^{th} root of a is x, then it is written as $x^n = a$ or $\sqrt[n]{a} = x$.

Surds

1. If a is a positive rational number and n^{th} root of a is x, and if x is an irrational number, then x is called a **surd**. (Surd is an irrational root).

   Examples:

   i. $\sqrt{5}$ is a surd.
   Here, 5 is a positive rational number, 2 is a positive integer greater than 1 and $\sqrt{5}$ is an irrational number.

   ii. $\sqrt[3]{8}$ is not a surd because $\sqrt[3]{8} = 2$, which is not an irrational number.

2. In a surd $\sqrt[n]{a}$ the symbol $\sqrt{\ }$ is called **radical sign**, n is the **order** of the surd and a is called **radicand**.

3. **Order of a Surd:**
   Surd $\sqrt[n]{a}$ is said to be of order 'n'.
   The surd of order 2 is called 'quadratic surd'.

   Examples:

   i. $\sqrt{3}$ is a surd of order 2.

   ii. $\sqrt[3]{7}$ is a surd of order 3.

   iii. $\sqrt[4]{15}$ is a surd of order 4.

Simplest form of a surd

Examples: i. $\sqrt{32} = \sqrt{16 \times 2} = \sqrt{4} \times \sqrt{2} = 2\sqrt{2}$

ii. $\sqrt{\dfrac{7}{9}} = \dfrac{\sqrt{7}}{\sqrt{9}} = \dfrac{\sqrt{7}}{3}$

Similar or like surds

The surds of the form $p\sqrt{a}$ and $q\sqrt{a}$, where 'p' and 'q' are rational numbers, are called similar surds or like surds.

Example: $\sqrt{3}$, $4\sqrt{3}$, $\dfrac{2}{5}\sqrt{3}$ are all like surds.

Remember This

i. In the simplest form of the surds if order of the surds and redicand are equal, then the surds are similar or like surds.

Comparison of surds

Similar surds can be compared by comparing the radicands.

If $\sqrt{a}$ and $\sqrt{b}$ are like surds, then

i. $\sqrt{a} = \sqrt{b}$ if $a = b$

ii. $\sqrt{a} > \sqrt{b}$ if $a > b$

iii. $\sqrt{a} < \sqrt{b}$ if $a < b$

Examples:

a. $\sqrt{5} < \sqrt{7}$ because $5 < 7$

b. $\sqrt{21} > \sqrt{15}$ because $21 > 15$

c. Compare $3\sqrt{5}$ and $\sqrt{7}$

Solution:

$3\sqrt{5} = \sqrt{9} \times \sqrt{5} = \sqrt{45}$

Here, $45 > 7$

$\therefore \quad \sqrt{45} > \sqrt{7}$

$\therefore \quad 3\sqrt{5} > \sqrt{7}$

Operations on like surds

Mathematical operations like addition, subtraction, multiplication and division can be done on like surds.

Examples:

i. $$3\sqrt{3} + 9\sqrt{3} + \frac{2}{3}\sqrt{3} = \left(3 + 9 + \frac{2}{3}\right)\sqrt{3}$$
$$= \left(\frac{9 + 27 + 2}{3}\right)\sqrt{3}$$
$$= \frac{38}{3}\sqrt{3}$$

$$\therefore \quad \mathbf{3\sqrt{3} + 9\sqrt{3} + \frac{2}{3}\sqrt{3} = \frac{38}{3}\sqrt{3}}$$

ii. $8\sqrt{5} - 4\sqrt{5} = (8 - 4)\sqrt{5} = \mathbf{4\sqrt{5}}$

iii. $5\sqrt{3} \times 7\sqrt{3} = 5 \times 7 \times \sqrt{3} \times \sqrt{3}$
$= 35 \times 3 = 105$

$\therefore \quad \mathbf{5\sqrt{3} \times 7\sqrt{3} = 105}$

iv. $$21\sqrt{8} \div 3\sqrt{2} = \frac{21\sqrt{8}}{3\sqrt{2}} = \frac{21 \times 2\sqrt{2}}{3\sqrt{2}} = 14$$

$\therefore \quad \mathbf{21\sqrt{8} \div 3\sqrt{2} = 14}$

Rationalization of surd

1. If the product of two surds is a rational number, then each surd is called the rationalizing factor of the other surd.

Example:

$\sqrt{27} \times \sqrt{3} = \sqrt{81} = 9$

$\therefore \quad \sqrt{3}$ is the rationalizing factor of $\sqrt{27}$.

2. **Rationalization of denominator:**
   It is convenient to have the denominator of any fraction as a real number. Rationalizing factor of the surd is used to rationalize the denominator.

Example: Rationalize the denominator $\frac{2}{\sqrt{5}}$.

Solution:

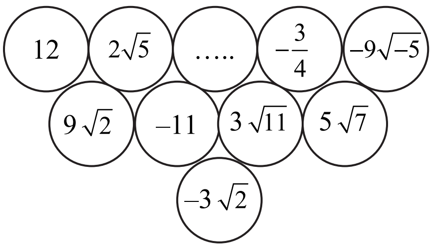
$$\frac{2}{\sqrt{5}} = \frac{2}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}}$$ …[Multiplying the numerator and denominator by $\sqrt{5}$]

$$\therefore \quad \frac{\mathbf{2}}{\sqrt{\mathbf{5}}} = \frac{\mathbf{2}\sqrt{\mathbf{5}}}{\mathbf{5}}$$

Try This

1. There are some real numbers written on a card sheet. Use these numbers and construct two examples each of addition, subtraction, multiplication and division. Solve these examples.

(Textbook pg. no. 34)



Ans: i. $9\sqrt{2} + 3\sqrt{2} = 12\sqrt{2}$

ii. $12 - 5\sqrt{7} = 12 - 5\sqrt{7}$

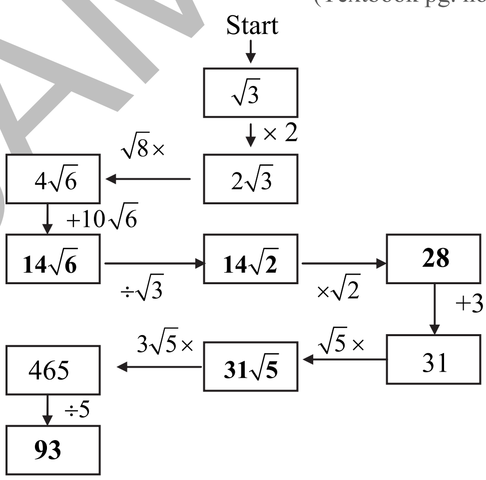
iii. $2\sqrt{5} \times 3\sqrt{11} = 6\sqrt{55}$

iv. $\frac{2\sqrt{5}}{9\sqrt{2}} = \frac{2\sqrt{5} \times \sqrt{2}}{9\sqrt{2} \times \sqrt{2}} = \frac{2\sqrt{10}}{9 \times 2} = \frac{\sqrt{10}}{9}$

[**Note:** Students should prepare other examples similar to the ones given and solve them.]

2. Follow the arrows and complete the chart by doing the operations given.

(Textbook pg. no. 34)



3. $\sqrt{\mathbf{9+16}}$ **?** $\sqrt{\mathbf{9}} + \sqrt{\mathbf{16}}$ (Textbook pg. no. 28)

Solution:

$\sqrt{9+16} = \sqrt{25} = 5$

$\sqrt{9} + \sqrt{16} = 3 + 4 = 7$

$\therefore \quad \sqrt{9+16} \neq \sqrt{9} + \sqrt{16}$

4. $\sqrt{\mathbf{100+36}}$ **?** $\sqrt{\mathbf{100}} + \sqrt{\mathbf{36}}$ (Textbook pg. no. 28)

Solution:

$\sqrt{100+36} = \sqrt{136} = 2\sqrt{34}$

$\sqrt{100} + \sqrt{36} = 10 + 6 = 16$

$\therefore \quad \sqrt{100+36} \neq \sqrt{100} + \sqrt{36}$

From the above examples,

$\sqrt{\mathbf{a+b}} \neq \sqrt{\mathbf{a}} + \sqrt{\mathbf{b}}$

Remember This

1. Rationalizing factor of a given surd is not unique.

Example: The rationalizing factors of $\sqrt{8}$ are $\sqrt{2}, 2\sqrt{2}, 5\sqrt{2}$, etc.

2. **Laws of Surds:**

If a, b $\in$ Q, a, b > 0 and m, n, p $\in$ N, then

i. $\left(\sqrt[n]{a}\right)^n = a$ ii. $\sqrt[n]{a} \cdot \sqrt[n]{b} = \sqrt[n]{ab}$

iii. $\frac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\frac{a}{b}}$ iv. $\sqrt[m]{\sqrt[n]{a}} = \sqrt[n]{\sqrt[m]{a}} = \sqrt[mn]{a}$

v. $\sqrt[m]{a^n} = \sqrt[mp]{a^{np}}$ vi. $\sqrt[m]{a^n} = \left(\sqrt[m]{a}\right)^n$

Practice Set 2.3

1. State the order of the surds given below.

i. $\sqrt[3]{7}$ ii. $5\sqrt{12}$

iii. $\sqrt[4]{10}$ iv. $\sqrt{39}$

v. $\sqrt[3]{18}$

Ans: i. 3 ii. 2

iii. 4 iv. 2

v. 3

2. State which of the following are surds. Justify.

i. $\sqrt[3]{51}$ ii. $\sqrt[4]{16}$

iii. $\sqrt[5]{81}$ iv. $\sqrt{256}$

v. $\sqrt[3]{64}$ vi. $\sqrt{\frac{22}{7}}$

Ans: i. $\sqrt[3]{\mathbf{51}}$ **is a surd** because 51 is a positive rational number, 3 is a positive integer greater than 1 and $\sqrt[3]{51}$ is irrational.

ii. $\sqrt[4]{16}$ **is not a surd** because $\sqrt[4]{16} = 2$, which is not an irrational number.

iii. $\sqrt[5]{81}$ **is a surd** because 81 is a positive rational number, 5 is a positive integer greater than 1 and $\sqrt[5]{81}$ is irrational.

iv. $\sqrt{256}$ **is not a surd** because $\sqrt{256} = 16$, which is not an irrational number.

v. $\sqrt[3]{64}$ **is not a surd** because $\sqrt[3]{64} = 4$, which is not an irrational number.

vi. $\sqrt{\frac{22}{7}}$ **is a surd** because $\frac{22}{7}$ is a positive rational number, 2 is a positive integer greater than 1 and $\sqrt{\frac{22}{7}}$ is irrational.

3. Classify the given pair of surds into like surds and unlike surds.

i. $\sqrt{52}, 5\sqrt{13}$ ii. $\sqrt{68}, 5\sqrt{3}$

iii. $4\sqrt{18}, 7\sqrt{2}$ iv. $19\sqrt{12}, 6\sqrt{3}$

v. $5\sqrt{22}, 7\sqrt{33}$ vi. $5\sqrt{5}, \sqrt{75}$

Solution:

i. $\sqrt{52} = \sqrt{4\times13} = 2\sqrt{13}$

$\therefore$ **$\sqrt{52}$ and $5\sqrt{13}$ are like surds.**

ii. $\sqrt{68} = \sqrt{4\times17} = 2\sqrt{17}$

$\therefore$ **$\sqrt{68}$ and $5\sqrt{3}$ are unlike surds.**

iii. $4\sqrt{18} = 4\times\sqrt{9\times2} = 4\times3\sqrt{2} = 12\sqrt{2}$

$\therefore$ **$4\sqrt{18}$ and $7\sqrt{2}$ are like surds.**

iv. $19\sqrt{12} = 19\times\sqrt{4\times3} = 19\times2\sqrt{3} = 38\sqrt{3}$

$\therefore$ **$19\sqrt{12}$ and $6\sqrt{3}$ are like surds.**

v. **$5\sqrt{22}$ and $7\sqrt{33}$ are unlike surds.**

vi. $\sqrt{75} = \sqrt{25\times3} = 5\sqrt{3}$

$\therefore$ **$5\sqrt{5}$ and $\sqrt{75}$ are unlike surds.**

4. Simplify the following surds.

i. $\sqrt{27}$ ii. $\sqrt{50}$

iii. $\sqrt{250}$ iv. $\sqrt{112}$

v. $\sqrt{168}$

Solution:

i. $\sqrt{27} = \sqrt{9\times3} = \mathbf{3\sqrt{3}}$

ii. $\sqrt{50} = \sqrt{25\times2} = \mathbf{5\sqrt{2}}$

iii. $\sqrt{250} = \sqrt{25\times10} = \mathbf{5\sqrt{10}}$

iv. $\sqrt{112} = \sqrt{16\times7} = \mathbf{4\sqrt{7}}$

v. $\sqrt{168} = \sqrt{4\times42} = \mathbf{2\sqrt{42}}$

5. Compare the following pair of surds.

i. $7\sqrt{2}, 5\sqrt{3}$ ii. $\sqrt{247}, \sqrt{274}$

iii. $2\sqrt{7}, \sqrt{28}$ iv. $5\sqrt{5}, 7\sqrt{2}$

v. $4\sqrt{42}, 9\sqrt{2}$ vi. $5\sqrt{3}, 9$

vii. $7, 2\sqrt{5}$

Solution:

i. $7\sqrt{2} = \sqrt{49}\times\sqrt{2} = \sqrt{98}$

$5\sqrt{3} = \sqrt{25}\times\sqrt{3} = \sqrt{75}$

Since, $98 > 75$

$\therefore \sqrt{98} > \sqrt{75}$

$\therefore$ $\mathbf{7\sqrt{2} > 5\sqrt{3}}$

ii. Since, $247 < 274$

$\therefore$ $\mathbf{\sqrt{247} < \sqrt{274}}$

iii. $2\sqrt{7} = \sqrt{4}\times\sqrt{7} = \sqrt{28}$

Since, $28 = 28$

$\therefore \sqrt{28} = \sqrt{28}$

$\therefore$ $\mathbf{2\sqrt{7} = \sqrt{28}}$

iv. $5\sqrt{5} = \sqrt{25}\times\sqrt{5} = \sqrt{125}$

$7\sqrt{2} = \sqrt{49}\times\sqrt{2} = \sqrt{98}$

Since, $125 > 98$

$\therefore \sqrt{125} > \sqrt{98}$

$\therefore$ $\mathbf{5\sqrt{5} > 7\sqrt{2}}$

v. $4\sqrt{42} = \sqrt{16}\times\sqrt{42} = \sqrt{672}$

$9\sqrt{2} = \sqrt{81}\times\sqrt{2} = \sqrt{162}$

Since, $672 > 162$

$\therefore \sqrt{672} > \sqrt{162}$

$\therefore$ $\mathbf{4\sqrt{42} > 9\sqrt{2}}$

vi. $5\sqrt{3} = \sqrt{25}\times\sqrt{3} = \sqrt{75}$

$9 = \sqrt{81}$

Since, $75 < 81$

$\therefore \sqrt{75} < \sqrt{81}$

$\therefore$ $\mathbf{5\sqrt{3} < 9}$

vii. $7 = \sqrt{49}$

$2\sqrt{5} = \sqrt{4}\times\sqrt{5} = \sqrt{20}$

Since, $49 > 20$

$\therefore \sqrt{49} > \sqrt{20}$

$\therefore$ $\mathbf{7 > 2\sqrt{5}}$

6. Simplify.

i. $5\sqrt{3} + 8\sqrt{3}$

ii. $9\sqrt{5} - 4\sqrt{5} + \sqrt{125}$

iii. $7\sqrt{48} - \sqrt{27} - \sqrt{3}$

iv. $\sqrt{7} - \frac{3}{5}\sqrt{7} + 2\sqrt{7}$

Solution:

i. $5\sqrt{3} + 8\sqrt{3} = (5+8)\sqrt{3} = 13\sqrt{3}$

$\therefore$ $\mathbf{5\sqrt{3} + 8\sqrt{3} = 13\sqrt{3}}$

---

ii. $9\sqrt{5} - 4\sqrt{5} + \sqrt{125}$

$= 9\sqrt{5} - 4\sqrt{5} + \sqrt{25\times5}$

$= 9\sqrt{5} - 4\sqrt{5} + 5\sqrt{5}$

$= (9 - 4 + 5)\sqrt{5}$

$= 10\sqrt{5}$

$\therefore$ $\mathbf{9\sqrt{5} - 4\sqrt{5} + \sqrt{125} = 10\sqrt{5}}$

---

iii. $7\sqrt{48} - \sqrt{27} - \sqrt{3}$

$= 7\sqrt{16\times3} - \sqrt{9\times3} - \sqrt{3}$

$= 7\times4\sqrt{3} - 3\sqrt{3} - \sqrt{3}$

$= 28\sqrt{3} - 3\sqrt{3} - \sqrt{3}$

$= (28 - 3 - 1)\sqrt{3}$

$= 24\sqrt{3}$

$\therefore$ $\mathbf{7\sqrt{48} - \sqrt{27} - \sqrt{3} = 24\sqrt{3}}$

---

iv. $\sqrt{7} - \frac{3}{5}\sqrt{7} + 2\sqrt{7}$

$= \left(1 - \frac{3}{5} + 2\right)\sqrt{7}$

$= \left(3 - \frac{3}{5}\right)\sqrt{7}$

$= \frac{12\sqrt{7}}{5}$

$\therefore$ $\mathbf{\sqrt{7} - \frac{3}{5}\sqrt{7} + 2\sqrt{7} = \frac{12\sqrt{7}}{5}}$

---

7. Multiply and write the answer in the simplest form.

i. $3\sqrt{12} \times \sqrt{18}$

ii. $3\sqrt{12} \times 7\sqrt{15}$

iii. $3\sqrt{8} \times \sqrt{5}$

iv. $5\sqrt{8} \times 2\sqrt{8}$

Solution:

i. $3\sqrt{12} \times \sqrt{18} = 3 \times \sqrt{4\times3}\times\sqrt{9\times2}$

$= 3 \times 2\sqrt{3} \times 3\sqrt{2}$

$= 18\sqrt{6}$

$\therefore$ $\mathbf{3\sqrt{12} \times \sqrt{18} = 18\sqrt{6}}$

ii. $3\sqrt{12} \times 7\sqrt{15} = 3 \times \sqrt{4\times3} \times 7 \times \sqrt{5\times3}$

$= 3 \times 2\sqrt{3} \times 7\sqrt{5} \times \sqrt{3}$

$= 42 \times 3 \times \sqrt{5}$

$= 126\sqrt{5}$

$\therefore$ $\mathbf{3\sqrt{12} \times 7\sqrt{15} = 126\sqrt{5}}$

---

iii. $3\sqrt{8} \times \sqrt{5} = 3 \times \sqrt{4\times2}\times\sqrt{5}$

$= 3\times2\sqrt{2}\times\sqrt{5}$

$= 6\sqrt{10}$

$\therefore$ $\mathbf{3\sqrt{8} \times \sqrt{5} = 6\sqrt{10}}$

---

iv. $5\sqrt{8} \times 2\sqrt{8} = 5 \times 2 \times 8$

$\therefore$ $\mathbf{5\sqrt{8} \times 2\sqrt{8} = 80}$

---

8. Divide and write the answer in simplest form.

i. $\sqrt{98} \div \sqrt{2}$ ii. $\sqrt{125} \div \sqrt{50}$

iii. $\sqrt{54} \div \sqrt{27}$ iv. $\sqrt{310} \div \sqrt{5}$

Solution:

i. $\frac{\sqrt{98}}{\sqrt{2}} = \sqrt{\frac{98}{2}} = \sqrt{49} = \mathbf{7}$

---

ii. $\frac{\sqrt{125}}{\sqrt{50}} = \sqrt{\frac{125}{50}} = \sqrt{\frac{25\times5}{25\times2}} = \mathbf{\sqrt{\frac{5}{2}}}$

---

iii. $\frac{\sqrt{54}}{\sqrt{27}} = \sqrt{\frac{54}{27}} = \mathbf{\sqrt{2}}$

---

iv. $\frac{\sqrt{310}}{\sqrt{5}} = \sqrt{\frac{310}{5}} = \mathbf{\sqrt{62}}$

---

9. Rationalize the denominator.

i. $\frac{3}{\sqrt{5}}$ ii. $\frac{1}{\sqrt{14}}$

iii. $\frac{5}{\sqrt{7}}$ iv. $\frac{6}{9\sqrt{3}}$

v. $\frac{11}{\sqrt{3}}$

Solution:

i. $\frac{3}{\sqrt{5}} = \frac{3}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}}$

...[Multiplying the numerator and denominator by $\sqrt{5}$]

$= \frac{3\times\sqrt{5}}{\sqrt{5}\times\sqrt{5}}$

$= \frac{3\sqrt{5}}{5}$

$\therefore$ $\mathbf{\frac{3}{\sqrt{5}} = \frac{3\sqrt{5}}{5}}$

ii. $\frac{1}{\sqrt{14}} = \frac{1}{\sqrt{14}} \times \frac{\sqrt{14}}{\sqrt{14}}$

...[Multiplying the numerator and denominator by $\sqrt{14}$]

$= \frac{1 \times \sqrt{14}}{\sqrt{14} \times \sqrt{14}}$

$= \frac{\sqrt{14}}{14}$

$\therefore$ $\mathbf{\frac{1}{\sqrt{14}} = \frac{\sqrt{14}}{14}}$

---

iii. $\frac{5}{\sqrt{7}} = \frac{5}{\sqrt{7}} \times \frac{\sqrt{7}}{\sqrt{7}}$

...[Multiplying the numerator and denominator by $\sqrt{7}$]

$= \frac{5 \times \sqrt{7}}{\sqrt{7} \times \sqrt{7}}$

$= \frac{5\sqrt{7}}{7}$

$\therefore$ $\mathbf{\frac{5}{\sqrt{7}} = \frac{5\sqrt{7}}{7}}$

---

iv. $\frac{6}{9\sqrt{3}} = \frac{6}{9\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$

...[Multiplying the numerator and denominator by $\sqrt{3}$]

$= \frac{6 \times \sqrt{3}}{9\sqrt{3} \times \sqrt{3}}$

$= \frac{6\sqrt{3}}{9 \times 3}$

$= \frac{2\sqrt{3}}{9}$

$\therefore$ $\mathbf{\frac{6}{9\sqrt{3}} = \frac{2\sqrt{3}}{9}}$

---

v. $\frac{11}{\sqrt{3}} = \frac{11}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$

...[Multiplying the numerator and denominator by $\sqrt{3}$]

$= \frac{11 \times \sqrt{3}}{\sqrt{3} \times \sqrt{3}}$

$= \frac{11\sqrt{3}}{3}$

$\therefore$ $\mathbf{\frac{11}{\sqrt{3}} = \frac{11\sqrt{3}}{3}}$

Let's Study

Binomial quadratic surd

1. The sum of two numbers, one of which is a quadratic surd and the other is either a non-zero rational number or a quadratic surd is called binomial quadratic surd.

Examples: $\sqrt{5} + \sqrt{3}$, $2 - \sqrt{5}$, $5 - 2\sqrt{11}$

2. Conjugate pair of binomial surds:

If the product of two binomial surds is a rational number, then the two numbers form a conjugate pair of surds.

Example: $\left(\sqrt{5}+\sqrt{3}\right)\left(\sqrt{5}-\sqrt{3}\right) = \left(\sqrt{5}\right)^2 - \left(\sqrt{3}\right)^2$

$= 25 - 9$

$= 16$

Hence, $\sqrt{5}+\sqrt{3}$ and $\sqrt{5}-\sqrt{3}$ are conjugate of each other.

Note: The conjugate of $\sqrt{5}+\sqrt{3}$ is $\sqrt{5}-\sqrt{3}$ or $-\sqrt{5}+\sqrt{3}$.

Rationalization of the denominator

The product of conjugate pair of binomial surds is always a rational number. By using this property, the rationalization of the denominator in the form of binomial surd can be done.

Example: Rationalize the denominator $\frac{1}{\sqrt{6}-\sqrt{2}}$.

Solution:

The conjugate of $\sqrt{6}-\sqrt{2}$ is $\sqrt{6}+\sqrt{2}$.

$\therefore$ $\frac{1}{\sqrt{6}-\sqrt{2}} = \frac{1}{\sqrt{6}-\sqrt{2}} \times \frac{\sqrt{6}+\sqrt{2}}{\sqrt{6}+\sqrt{2}}$

...[Multiplying the numerator and denominator by $\left(\sqrt{6}+\sqrt{2}\right)$]

$= \frac{\sqrt{6}+\sqrt{2}}{\left(\sqrt{6}\right)^2 - \left(\sqrt{2}\right)^2}$ ...[$\because (a + b)(a - b) = a^2 - b^2$]

$= \frac{\sqrt{6}+\sqrt{2}}{6-2}$

$= \mathbf{\frac{\sqrt{6}+\sqrt{2}}{4}}$

Practice Set 2.4

1. Multiply.

i. $\sqrt{3}\left(\sqrt{7}-\sqrt{3}\right)$

ii. $\left(\sqrt{5}-\sqrt{7}\right)\sqrt{2}$

iii. $\left(3\sqrt{2}-\sqrt{3}\right)\left(4\sqrt{3}-\sqrt{2}\right)$

***Solution*:**

i. $\sqrt{3}\left(\sqrt{7}-\sqrt{3}\right) = \sqrt{3}\times\sqrt{7} - \sqrt{3}\times\sqrt{3}$

$= \sqrt{21}-3$

$\therefore$ $\mathbf{\sqrt{3}\left(\sqrt{7}-\sqrt{3}\right) = -3+\sqrt{21}}$

---

ii. $\left(\sqrt{5}-\sqrt{7}\right)\sqrt{2} = \sqrt{5}\times\sqrt{2} - \sqrt{7}\times\sqrt{2}$

$\therefore$ $\mathbf{\left(\sqrt{5}-\sqrt{7}\right)\sqrt{2} = \sqrt{10}-\sqrt{14}}$

---

iii. $\left(3\sqrt{2}-\sqrt{3}\right)\left(4\sqrt{3}-\sqrt{2}\right)$

$= 3\sqrt{2}\left(4\sqrt{3}-\sqrt{2}\right) - \sqrt{3}\left(4\sqrt{3}-\sqrt{2}\right)$

$= 3\sqrt{2}\times 4\sqrt{3} - 3\sqrt{2}\times\sqrt{2} - \sqrt{3}\times 4\sqrt{3} + \sqrt{3}\times\sqrt{2}$

$= 12\sqrt{6} - 3\times 2 - 4\times 3 + \sqrt{6}$

$= 13\sqrt{6} - 6 - 12$

$= 13\sqrt{6} - 18$

$= -18 + 13\sqrt{6}$

$\therefore$ $\mathbf{\left(3\sqrt{2}-\sqrt{3}\right)\left(4\sqrt{3}-\sqrt{2}\right) = -18 + 13\sqrt{6}}$

---

2. Rationalize the denominator.

i. $\dfrac{1}{\sqrt{7}+\sqrt{2}}$ ii. $\dfrac{3}{2\sqrt{5}-3\sqrt{2}}$

iii. $\dfrac{4}{7+4\sqrt{3}}$ iv. $\dfrac{\sqrt{5}-\sqrt{3}}{\sqrt{5}+\sqrt{3}}$

***Solution*:**

i. $\dfrac{1}{\sqrt{7}+\sqrt{2}} = \dfrac{1}{\left(\sqrt{7}+\sqrt{2}\right)}\times\dfrac{\left(\sqrt{7}-\sqrt{2}\right)}{\left(\sqrt{7}-\sqrt{2}\right)}$

...[Multiplying the numerator and denominator by $\left(\sqrt{7}-\sqrt{2}\right)$]

$= \dfrac{\sqrt{7}-\sqrt{2}}{\left(\sqrt{7}\right)^2-\left(\sqrt{2}\right)^2}$

...[$\because$ $(a-b)(a+b) = a^2-b^2$]

$= \dfrac{\sqrt{7}-\sqrt{2}}{7-2}$

$\therefore$ $\mathbf{\dfrac{1}{\sqrt{7}+\sqrt{2}} = \dfrac{\sqrt{7}-\sqrt{2}}{5}}$

ii. $\dfrac{3}{2\sqrt{5}-3\sqrt{2}} = \dfrac{3}{\left(2\sqrt{5}-3\sqrt{2}\right)}\times\dfrac{\left(2\sqrt{5}+3\sqrt{2}\right)}{\left(2\sqrt{5}+3\sqrt{2}\right)}$

...[Multiplying the numerator and denominator by $\left(2\sqrt{5}+3\sqrt{2}\right)$]

$= \dfrac{3\left(2\sqrt{5}+3\sqrt{2}\right)}{\left(2\sqrt{5}\right)^2-\left(3\sqrt{2}\right)^2}$

...[$\because$ $(a-b)(a+b) = a^2-b^2$]

$= \dfrac{3\left(2\sqrt{5}+3\sqrt{2}\right)}{4\times 5-9\times 2}$

$= \dfrac{3\left(2\sqrt{5}+3\sqrt{2}\right)}{20-18}$

$\therefore$ $\mathbf{\dfrac{3}{2\sqrt{5}-3\sqrt{2}} = \dfrac{3\left(2\sqrt{5}+3\sqrt{2}\right)}{2}}$

---

iii. $\dfrac{4}{7+4\sqrt{3}} = \dfrac{4}{\left(7+4\sqrt{3}\right)}\times\dfrac{\left(7-4\sqrt{3}\right)}{\left(7-4\sqrt{3}\right)}$

...[Multiplying the numerator and denominator by $\left(7-4\sqrt{3}\right)$]

$= \dfrac{4\left(7-4\sqrt{3}\right)}{(7)^2-\left(4\sqrt{3}\right)^2}$

...[$\because$ $(a-b)(a+b) = a^2-b^2$]

$= \dfrac{4\left(7-4\sqrt{3}\right)}{49-16\times 3}$

$= \dfrac{4\left(7-4\sqrt{3}\right)}{49-48}$

$= \dfrac{4\left(7-4\sqrt{3}\right)}{1}$

$\therefore$ $\mathbf{\dfrac{4}{7+4\sqrt{3}} = 28-16\sqrt{3}}$

---

iv. $\dfrac{\sqrt{5}-\sqrt{3}}{\sqrt{5}+\sqrt{3}} = \dfrac{\left(\sqrt{5}-\sqrt{3}\right)}{\left(\sqrt{5}+\sqrt{3}\right)}\times\dfrac{\left(\sqrt{5}-\sqrt{3}\right)}{\left(\sqrt{5}-\sqrt{3}\right)}$

...[Multiplying the numerator and denominator by $\left(\sqrt{5}-\sqrt{3}\right)$]

$= \dfrac{\left(\sqrt{5}-\sqrt{3}\right)^2}{\left(\sqrt{5}\right)^2-\left(\sqrt{3}\right)^2}$

...[$\because$ $(a-b)(a+b) = a^2-b^2$]

$= \dfrac{\left(\sqrt{5}\right)^2-2\times\sqrt{5}\times\sqrt{3}+\left(\sqrt{3}\right)^2}{5-3}$

...[$\because$ $(a-b)^2 = a^2-2ab+b^2$]

$= \frac{5 - 2\sqrt{15} + 3}{2}$

$= \frac{8 - 2\sqrt{15}}{2}$

$= \frac{2\left(4 - \sqrt{15}\right)}{2}$

$= 4 - \sqrt{15}$

$\therefore \quad \frac{\sqrt{5} - \sqrt{3}}{\sqrt{5} + \sqrt{3}} = \mathbf{4 - \sqrt{15}}$

Let's Study

Absolute Value

1. If x is a real number then absolute value of x is its distance from zero on the number line which is written as $| x |$, and $| x |$ is read as 'absolute value of x' or 'modulus of x'.
2. i. If $x > 0$, then $|x| = x$.
   If x is positive then absolute value of x is x.
   Example: $|5| = 5$
   ii. If $x = 0$, then $|x| = 0$.
   If x is zero then absolute value of x is zero.
   Example: $|0| = 0$
   iii. If $x < 0$, then $|x| = -x$.
   If x is negative then its absolute value is opposite of x.
   Example: $|-3| = -(-3) = 3$
3. If $|x| = a$, then $x = \pm a$

Example: Solve $|3x + 1| = 6$

Solution:

$|3x + 1| = 6$

$\therefore \quad 3x + 1 = 6 \quad$ or $\quad 3x + 1 = -6$

$\therefore \quad 3x = 6 - 1 \quad$ or $\quad 3x = -6 - 1$

$\therefore \quad 3x = 5 \quad$ or $\quad 3x = -7$

$\therefore \quad x = \mathbf{\frac{5}{3}} \quad$ or $\quad x = \mathbf{\frac{-7}{3}}$

Remember This

i. The absolute value of any real number is never negative.

Practice Set 2.5

1. Find the value.

i. $|15 - 2|$ ii. $|4 - 9|$

iii. $|7| \times |-4|$

***Solution*:**

i. $|15 - 2| = |13| = \mathbf{13}$

ii. $|4 - 9| = |-5| = \mathbf{5}$

iii. $|7| \times |-4| = 7 \times 4 = \mathbf{28}$

2. Solve.

i. $|3x - 5| = 1$ ii. $|7 - 2x| = 5$

iii. $\left|\frac{8 - x}{2}\right| = 5$ iv. $\left|5 + \frac{x}{4}\right| = 5$

***Solution*:**

i. $|3x - 5| = 1$

$\therefore \quad 3x - 5 = 1 \quad$ or $\quad 3x - 5 = -1$

$\therefore \quad 3x = 1 + 5 \quad$ or $\quad 3x = -1 + 5$

$\therefore \quad 3x = 6 \quad$ or $\quad 3x = 4$

$\therefore \quad x = \mathbf{2} \quad$ or $\quad x = \mathbf{\frac{4}{3}}$

---

ii. $|7 - 2x| = 5$

$\therefore \quad 7 - 2x = 5 \quad$ or $\quad 7 - 2x = -5$

$\therefore \quad 7 - 5 = 2x \quad$ or $\quad 7 + 5 = 2x$

$\therefore \quad 2x = 2 \quad$ or $\quad 2x = 12$

$\therefore \quad x = \mathbf{1} \quad$ or $\quad x = \mathbf{6}$

---

iii. $\left|\frac{8 - x}{2}\right| = 5$

$\therefore \quad \frac{8 - x}{2} = 5 \quad$ or $\quad \frac{8 - x}{2} = -5$

$\therefore \quad 8 - x = 10 \quad$ or $\quad 8 - x = -10$

$\therefore \quad 8 - 10 = x \quad$ or $\quad 8 + 10 = x$

$\therefore \quad x = \mathbf{-2} \quad$ or $\quad x = \mathbf{18}$

---

iv. $\left|5 + \frac{x}{4}\right| = 5$

$\therefore \quad 5 + \frac{x}{4} = 5 \quad$ or $\quad 5 + \frac{x}{4} = -5$

$\therefore \quad \frac{x}{4} = 5 - 5 \quad$ or $\quad \frac{x}{4} = -5 - 5$

$\therefore \quad \frac{x}{4} = 0 \quad$ or $\quad \frac{x}{4} = -10$

$\therefore \quad x = \mathbf{0} \quad$ or $\quad x = \mathbf{-40}$

Problem Set – 2

1. Choose the correct alternative answer for the questions given below.

i. Which one of the following is an irrational number?

(A) $\sqrt{\frac{16}{25}}$ (B) $\sqrt{5}$

(C) $\frac{3}{9}$ (D) $\sqrt{196}$

ii. Which of the following is an irrational number?

(A) 0.17

(B) $1.\overline{513}$

(C) $0.27\overline{46}$

(D) 0.101001000.....

iii. Decimal expansion of which of the following is non-terminating recurring?

(A) $\frac{2}{5}$ (B) $\frac{3}{16}$

(C) $\frac{3}{11}$ (D) $\frac{137}{25}$

iv. Every point on the number line represent which of the following numbers?

(A) Natural numbers
(B) Irrational numbers
(C) Rational numbers
(D) Real numbers.

v. The number $0.\dot{4}$ in $\frac{p}{q}$ form is .....

(A) $\frac{4}{9}$ (B) $\frac{40}{9}$

(C) $\frac{3.6}{9}$ (D) $\frac{36}{9}$

vi. What is n , if n is not a perfect square number ?

(A) Natural number
(B) Rational number
(C) Irrational number
(D) Options A, B, C all are correct.

vii. Which of the following is not a surd ?

(A) $\sqrt{7}$ (B) $\sqrt[3]{17}$

(C) $\sqrt[3]{64}$ (D) $\sqrt{193}$

viii. What is the order of the surd $\sqrt[3]{\sqrt{5}}$?

(A) 3 (B) 2
(C) 6 (D) 5

ix. Which one is the conjugate pair of $2\sqrt{5}+\sqrt{3}$?

(A) $-2\sqrt{5}+\sqrt{3}$
(B) $-2\sqrt{5}-\sqrt{3}$
(C) $2\sqrt{3}-\sqrt{5}$
(D) $\sqrt{3}+2\sqrt{5}$

x. The value of $|12 - (13+7) \times 4|$ is _______.

(A) – 68 (B) 68
(C) – 32 (D) 32

Answers:

i.	(B)	ii.	(D)	iii.	(C)
iv.	(D)	v.	(A)	vi.	(C)
vii.	(C)	viii.	(C)	ix.	(A)
x	(B)				

Hints:

ii. Since the decimal expansion is neither terminating nor recurring, 0.101001000.... is an irrational number.

iii. $\frac{3}{11}$

$11 = 1 \times 11$

Since the denominator is not of the form $2^m \times 5^n$, the decimal expansion of $\frac{3}{11}$ will be non terminating recurring.

v. Let $x = 0.\dot{4}$

$\therefore$ $10x = 4.\dot{4}$

$\therefore$ $10x - x = 4.\dot{4} - 0.\dot{4}$

$\therefore$ $9x = 4$

$\therefore$ $x = \frac{4}{9}$

vii. $\sqrt[3]{64} = 4$, which is not an irrational number.

viii. $\sqrt[3]{\sqrt{5}} = \sqrt[3\times 2]{5} = \sqrt[6]{5}$

$\therefore$ Order = 6

ix. The conjugate of $2\sqrt{5} + \sqrt{3}$ is $2\sqrt{5} - \sqrt{3}$ or $-2\sqrt{5}+\sqrt{3}$

x. $|12 - (13+7) \times 4| = |12 - 20 \times 4|$
$= |12 - 80|$
$= |-68|$
$= 68$

2. Write the following numbers in $\frac{p}{q}$ form.

i. 0.555 ii. $29.\overline{568}$
iii. 9.315315..... iv. 357.417417.....
v. $30.\overline{219}$

***Solution*:**

i. $0.555 = \frac{0.555 \times 1000}{1 \times 1000} = \frac{555}{1000} = \frac{5 \times 111}{5 \times 200}$

$= \mathbf{\frac{111}{200}}$

ii. Let $x = 29.\overline{568}$ …(i)

Multiplying both sides by 1000,

$1000\,x = 29568.\overline{568}$ …(ii)

Subtracting (i) from (ii),

$1000x - x = 29568.\overline{568} - 29.\overline{568}$

$\therefore$ $999x = 29539$

$\therefore$ $x = \frac{29539}{999}$

$\therefore$ $\mathbf{29.\overline{568} = \frac{29539}{999}}$

iii. Let $x = 9.315315\ldots$

$\therefore$ $x = 9.\overline{315}$ …(i)

Multiplying both sides by 1000,

$1000x = 9315.\overline{315}$ …(ii)

Subtracting (i) from (ii),

$1000x - x = 9315.\overline{315} - 9.\overline{315}$

$\therefore$ $999x = 9306$

$\therefore$ $x = \dfrac{9306}{999} = \dfrac{9 \times 1034}{9 \times 111} = \dfrac{1034}{111}$

$\therefore$ $\mathbf{9.315315\ldots = \dfrac{1034}{111}}$

---

iv. Let $x = 357.417417\ldots$

$\therefore$ $x = 357.\overline{417}$ …(i)

Multiplying both sides by 1000,

$1000x = 357417.\overline{417}$ …(ii)

Subtracting (i) from (ii),

$1000x - x = 357417.\overline{417} - 357.\overline{417}$

$\therefore$ $999x = 357060$

$\therefore$ $x = \dfrac{357060}{999} = \dfrac{3 \times 119020}{3 \times 333}$

$\therefore$ $\mathbf{357.417417\ldots = \dfrac{119020}{333}}$

---

v. Let $x = 30.\overline{219}$ …(i)

Multiplying both sides by 1000,

$1000x = 30219.\overline{219}$ …(ii)

Subtracting (i) from (ii),

$1000x - x = 30219.\overline{219} - 30.\overline{219}$

$\therefore$ $999x = 30189$

$\therefore$ $x = \dfrac{30189}{999} = \dfrac{3 \times 10063}{3 \times 333}$

$\therefore$ $\mathbf{30.\overline{219} = \dfrac{10063}{333}}$

---

3. Write the following numbers in its decimal form.

i. $\dfrac{-5}{7}$ ii. $\dfrac{9}{11}$

iii. $\sqrt{5}$ iv. $\dfrac{121}{13}$

v. $\dfrac{29}{8}$

***Solution*:**

i. $\dfrac{-5}{7} = -0.714285714285\ldots..$

$\therefore$ $\mathbf{\dfrac{-5}{7} = -0.\overline{714285}}$

ii. $\dfrac{9}{11} = 0.818181\ldots..$

$\therefore$ $\mathbf{\dfrac{9}{11} = 0.\overline{81}}$

---

iii.

	2.23606
2	$\overline{5}.\overline{00}\,\overline{00}\,\overline{00}\,\overline{00}$....
+ 2	– 4
42	100
+ 2	– 84
443	1600
+ 3	– 1329
4466	27100
+ 6	– 26796
44720	30400
+ 0	– 0
447206	3040000
+ 6	– 2683236
447212	356764

$\therefore$ $\mathbf{\sqrt{5} = 2.23606\ldots..}$

---

iv. $\dfrac{121}{13} = 9.307692307692\ldots..$

$\therefore$ $\mathbf{\dfrac{121}{13} = 9.\overline{307692}}$

---

v. $\dfrac{29}{8} = \dfrac{29 \times 125}{8 \times 125} = \dfrac{3625}{1000} = \mathbf{3.625}$

---

4. Show that $5+\sqrt{7}$ is an irrational number.

***Solution*:**

Let us assume that $5+\sqrt{7}$ is a rational number.

So, we can find co-prime integers 'a' and 'b' (b ≠ 0) such that

$5+\sqrt{7} = \dfrac{a}{b}$

$\therefore$ $\sqrt{7} = \dfrac{a}{b} - 5$

Since, 'a' and 'b' are integers, $\dfrac{a}{b} - 5$ is a rational number and so $\sqrt{7}$ is a rational number.

But this contradicts the fact that $\sqrt{7}$ is an irrational number.

$\therefore$ Our assumption that $5+\sqrt{7}$ is a rational number is wrong.

$\therefore$ **$5+\sqrt{7}$ is an irrational number.**

5. Write the following surds in simplest form.

i. $\frac{3}{4}\sqrt{8}$ ii. $-\frac{5}{9}\sqrt{45}$

***Solution*:**

i. $\frac{3}{4}\sqrt{8} = \frac{3}{4} \times \sqrt{4\times2}$

$= \frac{3}{4}\times2\sqrt{2}$

$\therefore$ $\mathbf{\frac{3}{4}\sqrt{8} = \frac{3}{2}\sqrt{2}}$

ii. $-\frac{5}{9}\sqrt{45} = -\frac{5}{9}\times\sqrt{9\times5}$

$= -\frac{5}{9}\times 3\sqrt{5}$

$\therefore$ $\mathbf{-\frac{5}{9}\sqrt{45} = \frac{-5}{3}\sqrt{5}}$

---

6. Write the simplest form of rationalising factor for the given surds.

i. $\sqrt{32}$ ii. $\sqrt{50}$

iii. $\sqrt{27}$ iv. $\frac{3}{5}\sqrt{10}$

v. $3\sqrt{72}$ vi. $4\sqrt{11}$

***Solution*:**

i. $\sqrt{32} = \sqrt{16\times2} = 4\sqrt{2}$

Now, $4\sqrt{2} \times \sqrt{2} = 4 \times 2 = 8$, which is a rational number.

$\therefore$ **$\sqrt{2}$ is the simplest form of the rationalising factor of $\sqrt{32}$.**

ii. $\sqrt{50} = \sqrt{25\times2} = 5\sqrt{2}$

Now, $5\sqrt{2} \times \sqrt{2} = 5 \times 2 = 10$, which is a rational number.

$\therefore$ **$\sqrt{2}$ is the simplest form of the rationalising factor of $\sqrt{50}$.**

iii. $\sqrt{27} = \sqrt{9\times3} = 3\sqrt{3}$

Now, $3\sqrt{3} \times \sqrt{3} = 3 \times 3 = 9$, which is a rational number.

$\therefore$ **$\sqrt{3}$ is the simplest form of the rationalising factor of $\sqrt{27}$.**

iv. $\frac{3}{5}\sqrt{10} \times \sqrt{10} = \frac{3}{5}\times10 = 6$, which is a rational number.

$\therefore$ **$\sqrt{10}$ is the simplest form of the rationalising factor of $\frac{3}{5}\sqrt{10}$.**

v. $3\sqrt{72} = 3\sqrt{36\times2} = 3\times 6\sqrt{2} = 18\sqrt{2}$

Now, $18\sqrt{2}\times\sqrt{2} = 18 \times 2 = 36$, which is a rational number.

$\therefore$ **$\sqrt{2}$ is the simplest form of the rationalising factor of $3\sqrt{72}$.**

vi. $4\sqrt{11} \times \sqrt{11} = 4 \times 11 = 44$, which is a rational number.

$\therefore$ **$\sqrt{11}$ is the simplest form of the rationalising factor of $4\sqrt{11}$.**

---

7. Simplify.

i. $\frac{4}{7}\sqrt{147}+\frac{3}{8}\sqrt{192}-\frac{1}{5}\sqrt{75}$

ii. $5\sqrt{3}+2\sqrt{27}+\frac{1}{\sqrt{3}}$

iii. $\sqrt{216}-5\sqrt{6}+\sqrt{294}-\frac{3}{\sqrt{6}}$

iv. $4\sqrt{12}-\sqrt{75}-7\sqrt{48}$

v. $2\sqrt{48}-\sqrt{75}-\frac{1}{\sqrt{3}}$

***Solution*:**

i. $\frac{4}{7}\sqrt{147}+\frac{3}{8}\sqrt{192}-\frac{1}{5}\sqrt{75}$

$= \frac{4}{7}\sqrt{49\times3}+\frac{3}{8}\sqrt{64\times3}-\frac{1}{5}\sqrt{25\times3}$

$= \frac{4}{7}\times7\sqrt{3}+\frac{3}{8}\times8\sqrt{3}-\frac{1}{5}\times5\sqrt{3}$

$= 4\sqrt{3}+3\sqrt{3}-\sqrt{3}$

$= (4+3-1)\sqrt{3}$

$= 6\sqrt{3}$

$\therefore$ $\mathbf{\frac{4}{7}\sqrt{147}+\frac{3}{8}\sqrt{192}-\frac{1}{5}\sqrt{75} = 6\sqrt{3}}$

ii. $5\sqrt{3}+2\sqrt{27}+\frac{1}{\sqrt{3}}$

$= 5\sqrt{3}+2\sqrt{9\times3}+\frac{1\times\sqrt{3}}{\sqrt{3}\times\sqrt{3}}$

$= 5\sqrt{3}+2\times 3\sqrt{3}+\frac{\sqrt{3}}{3}$

$= 5\sqrt{3}+6\sqrt{3}+\frac{\sqrt{3}}{3}$

$= \left(5+6+\frac{1}{3}\right)\sqrt{3}$

$= \left(11+\frac{1}{3}\right)\sqrt{3}$

$= \frac{34}{3}\sqrt{3}$

$\therefore$ $\mathbf{5\sqrt{3}+2\sqrt{27}+\frac{1}{\sqrt{3}} = \frac{34}{3}\sqrt{3}}$

iii. $\sqrt{216}-5\sqrt{6}+\sqrt{294}-\frac{3}{\sqrt{6}}$

$= \sqrt{36\times6}-5\sqrt{6}+\sqrt{49\times6}-\frac{3\times\sqrt{6}}{\sqrt{6}\times\sqrt{6}}$

$= 6\sqrt{6}-5\sqrt{6}+7\sqrt{6}-\frac{3\sqrt{6}}{6}$

$= 6\sqrt{6}-5\sqrt{6}+7\sqrt{6}-\frac{1}{2}\sqrt{6}$

$= \left(6-5+7-\frac{1}{2}\right)\sqrt{6}$

$= \left(8-\frac{1}{2}\right)\sqrt{6}$

$= \frac{15}{2}\sqrt{6}$

$\therefore$ $\boldsymbol{\sqrt{216}-5\sqrt{6}+\sqrt{294}-\frac{3}{\sqrt{6}} = \frac{15}{2}\sqrt{6}}$

---

iv. $4\sqrt{12}-\sqrt{75}-7\sqrt{48}$

$= 4\sqrt{4\times3}-\sqrt{25\times3}-7\sqrt{16\times3}$

$= 4\times 2\sqrt{3}-5\sqrt{3}-7\times4\sqrt{3}$

$= 8\sqrt{3}-5\sqrt{3}-28\sqrt{3}$

$= (8-5-28)\sqrt{3}$

$= (-25)\sqrt{3}$

$\therefore$ $\boldsymbol{4\sqrt{12}-\sqrt{75}-7\sqrt{48} = -25\sqrt{3}}$

---

v. $2\sqrt{48}-\sqrt{75}-\frac{1}{\sqrt{3}}$

$= 2\sqrt{16\times3}-\sqrt{25\times3}-\frac{1\times\sqrt{3}}{\sqrt{3}\times\sqrt{3}}$

$= 2\times4\sqrt{3}-5\sqrt{3}-\frac{1}{3}\sqrt{3}$

$= 8\sqrt{3}-5\sqrt{3}-\frac{1}{3}\sqrt{3}$

$= \left(8-5-\frac{1}{3}\right)\sqrt{3}$

$= \left(3-\frac{1}{3}\right)\sqrt{3}$

$= \frac{8}{3}\sqrt{3}$

$\therefore$ $\boldsymbol{2\sqrt{48}-\sqrt{75}-\frac{1}{\sqrt{3}} = \frac{8}{3}\sqrt{3}}$

8. Rationalize the denominator.

i. $\frac{1}{\sqrt{5}}$

ii. $\frac{2}{3\sqrt{7}}$

iii. $\frac{1}{\sqrt{3}-\sqrt{2}}$

iv. $\frac{1}{3\sqrt{5}+2\sqrt{2}}$

v. $\frac{12}{4\sqrt{3}-\sqrt{2}}$

***Solution*:**

i. $\frac{1}{\sqrt{5}} = \frac{1}{\sqrt{5}}\times\frac{\sqrt{5}}{\sqrt{5}}$

...[Multiplying the numerator and denominator by $\sqrt{5}$]

$= \frac{1\times\sqrt{5}}{\sqrt{5}\times\sqrt{5}}$

$\therefore$ $\boldsymbol{\frac{1}{\sqrt{5}} = \frac{\sqrt{5}}{5}}$

---

ii. $\frac{2}{3\sqrt{7}} = \frac{2}{3\sqrt{7}}\times\frac{\sqrt{7}}{\sqrt{7}}$

...[Multiplying the numerator and denominator by $\sqrt{7}$]

$= \frac{2\times\sqrt{7}}{3\sqrt{7}\times\sqrt{7}} = \frac{2\sqrt{7}}{3\times7}$

$\therefore$ $\boldsymbol{\frac{2}{3\sqrt{7}} = \frac{2\sqrt{7}}{21}}$

---

iii. $\frac{1}{\sqrt{3}-\sqrt{2}} = \frac{1}{\left(\sqrt{3}-\sqrt{2}\right)}\times\frac{\left(\sqrt{3}+\sqrt{2}\right)}{\left(\sqrt{3}+\sqrt{2}\right)}$

...[Multiplying the numerator and denominator by $\left(\sqrt{3}+\sqrt{2}\right)$]

$= \frac{1\times\left(\sqrt{3}+\sqrt{2}\right)}{\left(\sqrt{3}-\sqrt{2}\right)\left(\sqrt{3}+\sqrt{2}\right)}$

$= \frac{\sqrt{3}+\sqrt{2}}{\left(\sqrt{3}\right)^2-\left(\sqrt{2}\right)^2}$

...[$\because$ $(a+b)(a-b) = a^2-b^2$]

$= \frac{\sqrt{3}+\sqrt{2}}{3-2}$

$\therefore$ $\boldsymbol{\frac{1}{\sqrt{3}-\sqrt{2}} = \sqrt{3}+\sqrt{2}}$

iv. $\dfrac{1}{3\sqrt{5}+2\sqrt{2}} = \dfrac{1}{\left(3\sqrt{5}+2\sqrt{2}\right)} \times \dfrac{\left(3\sqrt{5}-2\sqrt{2}\right)}{\left(3\sqrt{5}-2\sqrt{2}\right)}$

...[Multiplying the numerator and denominator by $\left(3\sqrt{5}-2\sqrt{2}\right)$]

$= \dfrac{1\times\left(3\sqrt{5}-2\sqrt{2}\right)}{\left(3\sqrt{5}+2\sqrt{2}\right)\left(3\sqrt{5}-2\sqrt{2}\right)}$

$= \dfrac{3\sqrt{5}-2\sqrt{2}}{\left(3\sqrt{5}\right)^2-\left(2\sqrt{2}\right)^2}$

...[$\because$ (a + b)(a – b) = $a^2 - b^2$]

$= \dfrac{3\sqrt{5}-2\sqrt{2}}{9\times5-4\times2}$

$= \dfrac{3\sqrt{5}-2\sqrt{2}}{45-8}$

$\therefore$ $\dfrac{\mathbf{1}}{\mathbf{3\sqrt{5}+2\sqrt{2}}} = \dfrac{\mathbf{3\sqrt{5}-2\sqrt{2}}}{\mathbf{37}}$

v. $\dfrac{12}{4\sqrt{3}-\sqrt{2}} = \dfrac{12}{\left(4\sqrt{3}-\sqrt{2}\right)} \times \dfrac{\left(4\sqrt{3}+\sqrt{2}\right)}{\left(4\sqrt{3}+\sqrt{2}\right)}$

...[Multiplying the numerator and denominator by $\left(4\sqrt{3}+\sqrt{2}\right)$]

$= \dfrac{12\left(4\sqrt{3}+\sqrt{2}\right)}{\left(4\sqrt{3}-\sqrt{2}\right)\left(4\sqrt{3}+\sqrt{2}\right)}$

$= \dfrac{12\left(4\sqrt{3}+\sqrt{2}\right)}{\left(4\sqrt{3}\right)^2-\left(\sqrt{2}\right)^2}$

...[$\because$ (a + b)(a – b) = $a^2 - b^2$]

$= \dfrac{12\left(4\sqrt{3}+\sqrt{2}\right)}{16\times3-2} = \dfrac{12\left(4\sqrt{3}+\sqrt{2}\right)}{48-2}$

$= \dfrac{12\left(4\sqrt{3}+\sqrt{2}\right)}{46}$

$\therefore$ $\dfrac{\mathbf{12}}{\mathbf{4\sqrt{3}-\sqrt{2}}} = \dfrac{\mathbf{6\left(4\sqrt{3}+\sqrt{2}\right)}}{\mathbf{23}}$

Multiple Choice Questions

1. The decimal form of which of the following rational number will be of terminating type?
   (A) $\dfrac{49}{6}$ (B) $\dfrac{800}{12}$
   (C) $\dfrac{67}{30}$ (D) $\dfrac{97}{8}$

2. The $\dfrac{p}{q}$ form of the recurring decimal $8.686686....$ is _______.
   (A) $\dfrac{86}{99}$ (B) $\dfrac{8678}{999}$
   (C) $\dfrac{8686}{999}$ (D) $\dfrac{78}{99}$

3. To convert $8.38\dot{7}$ in $\dfrac{p}{q}$ form, we will have to multiply $8.38\dot{7}$ by _______.
   (A) 10 (B) 100
   (C) 1000 (D) 10000

4. Which of the following is not an irrational number?
   (A) $\sqrt{5}$ (B) $\sqrt{8}$
   (C) $\sqrt{6}+2$ (D) $\sqrt{16}+3$

5. Square root of a negative number is _______.
   (A) a real number
   (B) an irrational number
   (C) not a real number
   (D) a negative number

6. Which of the following options are not of like surds?
   (A) $\sqrt{7}$, $\dfrac{1}{8}\sqrt{175}$, $-6\sqrt{343}$
   (B) $5\sqrt{5}$, $6\sqrt{35}$, $\sqrt{125}$
   (C) $\sqrt{11}$, $\sqrt{396}$, $\dfrac{5}{2}\sqrt{11}$
   (D) $\sqrt{8}$, $\sqrt{288}$, $\sqrt{968}$

7. The rationalizing factor of $\sqrt{80}$ is _______.
   (A) $\sqrt{2}$ (B) $\sqrt{3}$
   (C) $\sqrt{5}$ (D) $\sqrt{10}$

8. The order of the surd $\sqrt[6]{8}$ is _______.
   (A) 6 (B) $\dfrac{1}{6}$
   (C) 8 (D) $\dfrac{1}{8}$

9. The conjugate of the surd $7+\sqrt{6}$ is _______.
   (A) $7+\sqrt{6}$
   (B) $7-\sqrt{6}$
   (C) $-7-\sqrt{6}$
   (D) none of these

10. The numerator and denominator of $\frac{\sqrt{7}+\sqrt{6}}{\sqrt{7}-\sqrt{6}}$ will have to be multiplied by _______ to rationalise the denominator.
(A) $\sqrt{7}+\sqrt{6}$ (B) $\sqrt{7}-\sqrt{6}$
(C) $-\sqrt{7}+\sqrt{6}$ (D) none of these

11. The absolute value of a real number is always _______.
(A) positive
(B) negative
(C) positive or negative
(D) cannot be predicted

Additional Problems for Practice

Based on Practice Set 2.1

1. Write the following rational numbers in decimal form:
i. $\frac{1}{5}$ ii. $\frac{17}{99}$
iii. $\frac{223}{400}$

2. Write the following decimal numbers in $\frac{p}{q}$ form:
i. $0.\overline{18}$ ii. $0.\overline{513}$
iii. $4.\overline{7}$ iv. $7.\overline{529}$
+v. 0.777..... +vi. 7.529529529...

Based on Practice Set 2.2

1. Show $\sqrt{7}$, $-\sqrt{5}$ on number line.
2. Show that $6+\sqrt{7}$ is an irrational number.
+3. Show that $\sqrt{2}$ is an irrational number.

Based on Practice Set 2.3

1. State which of the following are surds. Justify.
i. $\sqrt{961}$ ii. $\sqrt{-6}$
iii. $\frac{1}{\sqrt{15}}$ iv. $3\sqrt{17}$
+v. $\sqrt[3]{8}$ +vi. $\sqrt[4]{8}$

2. Compare the following pair of surds.
i. $5\sqrt{6}, 6\sqrt{5}$ ii. $4\sqrt{7}, 5\sqrt{2}$
iii. $3\sqrt{17}, 19\sqrt{2}$ +iv. $6\sqrt{2}, 5\sqrt{5}$
+v. $8\sqrt{3}, \sqrt{192}$ +vi. $7\sqrt{2}, 5\sqrt{3}$

3. Simplify the following surds.
i. $6\sqrt{32}-8\sqrt{72}+\sqrt{242}-\sqrt{2}$
ii. $\frac{1}{4}\sqrt{243}+\sqrt{\frac{27}{4}}$
iii. $\sqrt{5}-\frac{7}{2}\sqrt{80}+\frac{11}{4}\sqrt{720}$
+iv. $7\sqrt{3}+29\sqrt{3}$
+v. $7\sqrt{3}-29\sqrt{3}$
+vi. $13\sqrt{8}+\frac{1}{2}\sqrt{8}-5\sqrt{8}$
+vii. $8\sqrt{5}+\sqrt{20}-\sqrt{125}$

4. Rationalize the denominator.
i. $\frac{6}{\sqrt{3}}$ ii. $\frac{-5}{2\sqrt{5}}$
iii. $\frac{\sqrt{17}}{\sqrt{2}}$ iv. $\frac{5\sqrt{2}}{3\sqrt{3}}$
+v. $\frac{1}{\sqrt{5}}$ +vi. $\frac{3}{2\sqrt{7}}$

+5. Write the simplest form of the surds.
i. $\sqrt{48}$ ii. $\sqrt{98}$

+6. Multiply the surds.
i. $\sqrt{7}\times\sqrt{42}$ ii. $\sqrt{50}\times\sqrt{18}$

+7. Divide the surds : $\sqrt{125}\div\sqrt{5}$

+8. Find the rationalizing factor of $\sqrt{27}$

Based on Practice Set 2.4

1. Rationalize the denominator.
i. $\frac{6}{2\sqrt{3}+\sqrt{6}}$ ii. $\frac{5\sqrt{2}}{7-\sqrt{2}}$
iii. $\frac{2\sqrt{5}-\sqrt{2}}{2\sqrt{5}+\sqrt{2}}$ iv. $\frac{3\sqrt{2}+2\sqrt{3}}{4\sqrt{2}-3\sqrt{3}}$
+v. $\frac{1}{\sqrt{5}-\sqrt{3}}$ +vi. $\frac{8}{3\sqrt{2}+\sqrt{5}}$

Based on Practice Set 2.5

1. Find the value.
i. $|3-5|$ ii. $|15|+|-15|$
iii. $-|3|\times|7|$ iv. $|-3|\times|7|$
+v. $|3|$ +vi. $|-3|$
+vii. $|0|$ +viii. $|9-5|$
+ix. $|8-13|$ +x. $|8|-|-3|$
+xi. $|8|\times|4|$

2. Solve.
+i. $|x-5|=2$ ii. $|3x-5|=1$
iii. $\left|5-\frac{1}{2}x\right|=\frac{1}{4}$

Apply your knowledge

1. Draw three or four circles of different radii on a card board. Cut these circles. Take a thread and measure the length of circumference and diameter of each of the circles. Note down the readings in the given table.

No.	radius (r)	diameter (d)	Circum-ference (c)	Ratio = $\frac{c}{d}$
i.	7 cm			
ii.	8 cm			
iii.	5.5 cm			

(Textbook pg. no. 23)

Ans:

i. 14, 44, 3.1   ii. 16, 50.3, 3.1
iii. 11, 34.6, 3.1

From table, we observe that the ratio $\frac{c}{d}$ is nearly 3.1 which is constant. This ratio is denoted by π (pi).

2. To find the approximate value of π, take the wire of length 11 cm, 22 cm and 33 cm each. Make a circle from the wire. Measure the diameter and complete the following table.

Circle No.	Circum-ference (c)	Diameter (d)	Ratio of (c) to (d)
i.	11 cm		
ii.	22 cm		
iii.	33 cm		

Verify that the ratio of circumference to the diameter of a circle is approximately $\frac{22}{7}$.

(Textbook pg. no. 24)

Ans:

i. 3.5, $\frac{22}{7}$ ii. 7, $\frac{22}{7}$

iii. 10.5, $\frac{22}{7}$

$\therefore$ The ratio of circumference to the diameter of each circle is $\frac{22}{7}$.

Practice Test

Total marks: 25

1. Write the correct alternative answer for each of the following questions. [5]

i. A rational number $\frac{p}{q}$ will be terminating decimal type, only when prime factors of q are ________ .
(A) 2 or 3   (B) 2 or 7
(C) 2 and 5   (D) 5 and 3

ii. $\frac{p}{q}$ form of 0.3333.... is
(A) $\frac{7}{3}$ (B) $\frac{3}{7}$
(C) $\frac{1}{3}$ (D) $\frac{3}{1}$

iii. Simplest form of surd $\sqrt[4]{48}$ is ________.
(A) $2\sqrt[4]{6}$ (B) $4\sqrt[4]{3}$
(C) $2\sqrt[4]{3}$ (D) $2\sqrt[4]{2}$

iv. Which of the following is a surd?
(A) $\sqrt{-5}$ (B) $\sqrt{484}$
(C) $\sqrt{\frac{5}{2}}$ (D) $\sqrt{121}$

v. The order of the surd $\sqrt[3]{\sqrt[4]{70}}$ is
(A) 3   (B) 4
(C) 7   (D) 12

2. Attempt the following. [3]

i. Write whether the decimal form of $\frac{7}{8}$ would be terminating or non-terminating recurring type.

ii. Write if the given pair of surds are like or unlike. $\sqrt{20}$, $3\sqrt{5}$

iii. Multiply: $10\sqrt{3} \times 3\sqrt{3}$

3. Attempt any three of the following. [6]

i. Compare the surds: $2\sqrt{6}$, $4\sqrt{2}$

ii. Simplify the surd $\frac{-7}{4}\sqrt{10}$

iii. Show $\sqrt{10}$ on number line.

iv. Write the decimal $1.\dot{6}$ in $\frac{p}{q}$ form.

4. Simplify any two of the following. **[6]**

i. $4\sqrt{8} + \sqrt{32} - \frac{10}{\sqrt{2}}$

ii. $\frac{5}{4}\sqrt{48} - \frac{3}{8}\sqrt{192} + \frac{1}{9}\sqrt{243}$

iii. $3\sqrt{7} + 7\sqrt{63} + \frac{1}{\sqrt{7}}$

5. Rationalize the denominator.

i. $\frac{5}{\sqrt{7}-\sqrt{5}}$ **[2]**

ii. $\frac{7\sqrt{3}-5\sqrt{2}}{\sqrt{48}+\sqrt{18}}$ **[3]**

Answers

Multiple Choice Questions

1. (D) 2. (B) 3. (C) 4. (D) 5. (C) 6. (B) 7. (C) 8. (A) 9. (B) 10. (A)
11. (A)

Additional Problems for Practice

Based on Practice Set 2.1

1. i. 0.2 ii. $0.\overline{17}$ iii. 0.5575

2. i. $\frac{2}{11}$ ii. $\frac{19}{37}$ iii. $\frac{43}{9}$ iv. $\frac{7522}{999}$
   v. $\frac{7}{9}$ vi. $\frac{7522}{999}$

Based on Practice Set 2.3

1. Surds: iii, iv, v, vi

2. i. $5\sqrt{6} < 6\sqrt{5}$ ii. $4\sqrt{7} > 5\sqrt{2}$ iii. $3\sqrt{17} < 19\sqrt{2}$ iv. $6\sqrt{2} < 5\sqrt{5}$
   v. $8\sqrt{3} = \sqrt{198}$ vi. $7\sqrt{2} > 5\sqrt{3}$

3. i. $-14\sqrt{2}$ ii. $\frac{15}{4}\sqrt{3}$ iii. $20\sqrt{5}$ iv. $36\sqrt{3}$
   v. $-22\sqrt{3}$ vi. $17\sqrt{2}$ vii. $5\sqrt{5}$

4. i. $2\sqrt{3}$ ii. $\frac{-\sqrt{5}}{2}$ iii. $\frac{\sqrt{34}}{2}$ iv. $\frac{5\sqrt{6}}{9}$
   v. $\frac{\sqrt{5}}{5}$ vi. $\frac{3\sqrt{7}}{14}$

5. i. $4\sqrt{3}$ ii. $7\sqrt{2}$

6. i. $7\sqrt{6}$ ii. 30

7. 5

8. $\sqrt{3}$

Based on Practice Set 2.4

1. i. $2\sqrt{3}-\sqrt{6}$ ii. $\frac{5\sqrt{2}(7+\sqrt{2})}{47}$ iii. $\frac{11-2\sqrt{10}}{9}$ iv. $\frac{42+17\sqrt{6}}{5}$
   v. $\frac{\sqrt{5}+\sqrt{3}}{2}$ vi. $\frac{24\sqrt{2}-8\sqrt{5}}{13}$

Based on Practice Set 2.5

1. i. 2 ii. 30 iii. –21 iv. 21
   v. 3 vi. 3 vii. 0 viii. 4
   ix. 5 x. 5 xi. 32
2. i. 7, 3 ii. 2, $\frac{4}{3}$ iii. $\frac{21}{2},\frac{19}{2}$

Practice Test

1. i. C ii. C iii. C iv. C
   v. D
2. i. Terminating ii. Like surds iii. 90
3. i. $2\sqrt{6} < 4\sqrt{2}$ ii. $-\sqrt{\frac{245}{8}}$ iv. $\frac{5}{3}$
4. i. $7\sqrt{2}$ ii. $3\sqrt{3}$ iii. $\frac{169}{7}\sqrt{7}$
5. i. $\frac{5}{2}\left(\sqrt{7}+\sqrt{5}\right)$ ii. $\frac{114-41\sqrt{6}}{30}$