Written as per the revised syllabus prescribed by the Maharashtra State Board
of Secondary and Higher Secondary Education, Pune.

STD. XI Sci.
Perfect Mathematics - I

Fifth Edition: May 2015

Salient Features

- Exhaustive coverage of entire syllabus.
- Covers answers to all textual and miscellaneous exercises.
- Precise theory for every topic.
- Neat, labelled and authentic diagrams.
- Written in systematic manner.
- Self evaluative in nature.
- Practice problems and multiple choice questions for effective preparation.

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Preface

In the case of good books, the point is not how many of them you can get through, but rather how many can get through to you.

"Std. XI Sci. : PERFECT MATHEMATICS - I" is a complete and thorough guide critically analysed and extensively drafted to boost the students confidence. The book is prepared as per the Maharashtra State board syllabus and provides answers to all **textual questions**. At the beginning of every chapter, topic – wise distribution of all textual questions including practice problems has been provided for simpler understanding of different types of questions. Neatly labelled diagrams have been provided wherever required.

Practice Problems and **Multiple Choice Questions** help the students to test their range of preparation and the amount of knowledge of each topic. Important theories and formulae are the highlights of this book. The steps are written in systematic manner for easy and effective understanding.

The journey to create a complete book is strewn with triumphs, failures and near misses. If you think we've nearly missed something or want to applaud us for our triumphs, we'd love to hear from you.

Please write to us on : mail@targetpublications.org

Best of luck to all the aspirants!

Yours faithfully,
Publisher

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Syllabus:
Directed angles, zero angle, straight angle, coterminal angles, standard angles, angle in a quadrant and quadrantal angles.

Systems of measurement of angles:
Sexagesimal system (degree measure), Circular system (radian measure), Relation between degree measure and radian measure, length of an arc of a circle and area of sector of a circle.

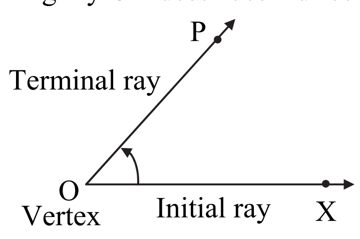
Introduction

In school geometry we have studied the definition of angle and trigonometric ratios of some acute angles. In this chapter we will extend the concept for different angles.

You are familiar with the definition of angle as the "union of two non-collinear rays having common end point". But according to this definition measure of angle is always positive and it lies between 0° to 180°. In order to study the concept of angle in broader manner, we will extend it for magnitude and sign.
The measurement of angle and sides of a triangle and the inter-relation between them was first studied by Greek astronomers Hipparchus and Ptolemy and Indian mathematicians Aryabhatta and Brahmagupta.

Directed angles

Suppose OX is the initial position of a ray. This ray rotates about O from initial position OX and takes a finite position along ray OP. In such a case we say that rotating ray OX describes a directed angle XOP.



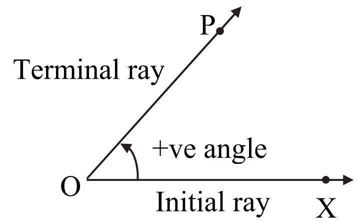
In the above figure, the point O is called the **vertex**. The ray OX is called the **initial ray** and ray OP is called the **terminal ray** of an angle XOP. The pair of rays are also called the **arms** of angle XOP.

In general, an angle can be defined as the ordered pair of initial and terminal rays or arms rotating from initial position to terminal position.

The directed angle includes two things
i. Amount of rotation (magnitude of angle).
ii. Direction of rotation (sign of the angle).

Positive angle:

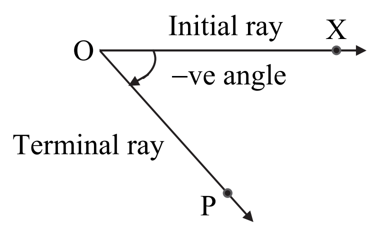
If a ray rotates about the vertex (the point) O from initial position OX in anticlockwise direction, then the angle described by the ray is positive angle.



In the above figure, ∠XOP is obtained by the rotation of a ray in anticlockwise direction denoted by arrow. Hence ∠XOP is positive i.e., +∠XOP.

Negative angle:

If a ray rotates about the vertex (the point) O, from initial position OX in clockwise direction, then the angle described by the ray is negative angle.



In the above figure, ∠XOP is obtained by the rotation of a ray in clockwise direction denoted by arrow. Hence ∠XOP is negative angle i.e., −∠XOP.

Angle of any magnitude:

i. Suppose a ray starts from the initial position OX in anticlockwise sense and makes complete rotation (revolution) about O and takes the final position along OX as shown in the figure (i), then the angle described by the ray is 360°.

Fig. (i)

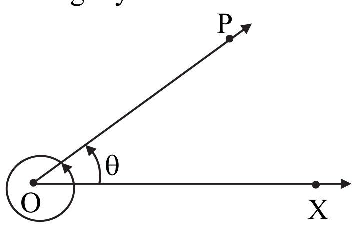
In figure (ii) initial ray rotates about O in anticlockwise sense and completes two rotations (revolutions). Hence, the angle described by the ray is 2 × 360° = 720°.

Fig. (ii)

In figure (iii) initial ray rotates about O in clockwise sense and completes two rotations (revolutions). Hence, the angle described by the ray is $-2 \times 360° = -720°$

Fig. (iii)

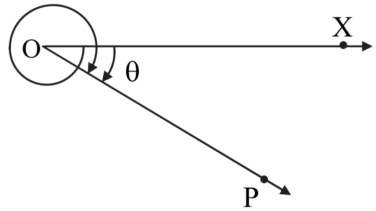
ii. Suppose a ray starting from the initial position OX makes one complete rotation in anticlockwise sense and takes the position OP as shown in figure, then the angle described by the revolving ray is $360° + \angle XOP$.



If $\angle XOP = \theta$, then the traced angle is $360° + \theta$.

If the rotating ray completes two rotations, then the angle described is $2 \times 360° + \theta = 720° + \theta$ and so on.

iii. Suppose the initial ray makes one complete rotation about O in clockwise sense and attains its terminal position OP, then the described angle is $-(360° + \angle XOP)$.



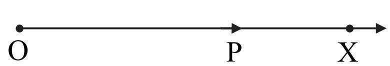
If $\angle XOP = \theta$, then the traced angle is $-(360° + \theta)$.

If final position OP is obtained after 2,3,4, …. complete rotations in clockwise sense, then angle described are $-(2 \times 360° + \theta)$, $-(3 \times 360° + \theta)$, $-(4 \times 360° + \theta)$, …..

Types of angles

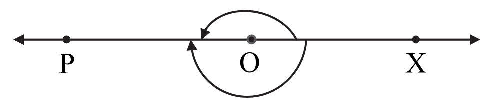
Zero angle:

If the initial ray and the terminal ray lie along same line and same direction i.e., they coincide, the angle so obtained is of measure zero and is called zero angle.



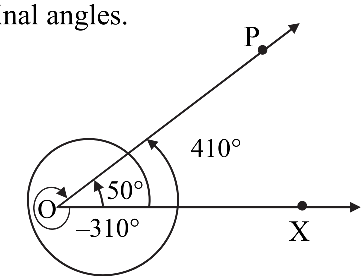
Straight angle:

In figure, OX is the initial position and OP is the final position of rotating ray. The rays OX and OP lie along the same line but in opposite direction. In this case $\angle XOP$ is called a straight angle and $m\angle XOP = 180°$.



Coterminal angles:

Two angles with different measures but having the same positions of initial and terminal ray are called as coterminal angles.



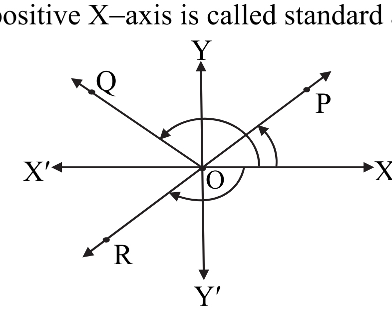
In figure, the directed angles having measures 50°, 410°, –310° have the same initial arm, ray OX and the same terminal arm, ray OP. Hence, these angles are coterminal angles.

Note:

If two directed angles are co-terminal angles, then the difference between measures of these two directed angles is an integral multiple of 360°.

Standard angle:

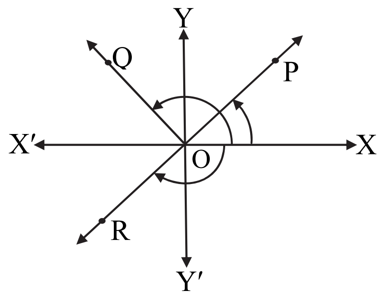
An angle which has vertex at origin and initial arm along positive X–axis is called standard angle.



In figure $\angle XOP$, $\angle XOQ$, $\angle XOR$ with vertex O and initial ray along positive X–axis are called standard angles or angles in standard position.

Angle in a Quadrant:

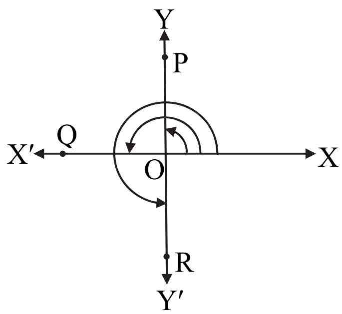
An angle is said to be in a particular quadrant, if the terminal ray of the angle in standard position lies in that quadrant.



In figure ∠XOP, ∠XOQ and ∠XOR lie in first, second and third quadrants respectively.

Quadrantal Angles:

If the terminal arm of an angle in standard position lie along any one of the co-ordinate axes, then it is called as quadrantal angle.



In figure ∠XOP, ∠XOQ, and ∠XOR are quadrantal angles.

Note:

The quadrantal angles are integral multiples of 90° i.e., $\pm n\frac{\pi}{2}$, where $n \in N$.

Systems of measurement of angles

There are two systems of measurement of an angle:

i. Sexagesimal system (Degree measure)

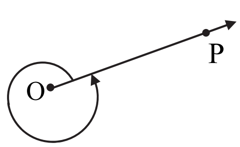
ii. Circular system (Radian measure)

i. Sexagesimal system (Degree Measure):

In this system, the unit of measurement of an angle is a degree.

Suppose a ray OP starts rotating in the anticlockwise sense about O and attains the original position for the first time, then the amount of rotation caused is called 1 revolution.

Divide 1 revolution into 360 equal parts. Each part is called as a one degree(1°).



1 revolution

i.e., 1 revolution = 360°

Divide 1° into 60 equal parts. Each part is called as a one minute (1′).

i.e., $1° = 60′$

Divide 1′ into 60 equal parts. Each part is called as a one second (1″)

i.e., $1′ = 60″$

Note:

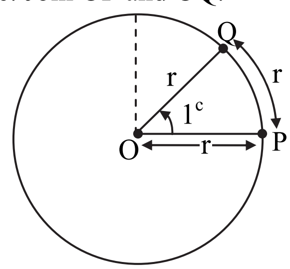
The sexagesimal system is extensively used in engineering, astronomy, navigation and surveying.

ii. Circular system (Radian measure):

In this system, the unit of measurement of an angle is radian.

Angle subtended at the centre of a circle by an arc whose length is equal to the radius is called as one radian denoted by 1^c.

Draw any circle with centre O and radius r. Take the points P and Q on the circle such that the length of arc PQ is equal to radius of the circle. Join OP and OQ.

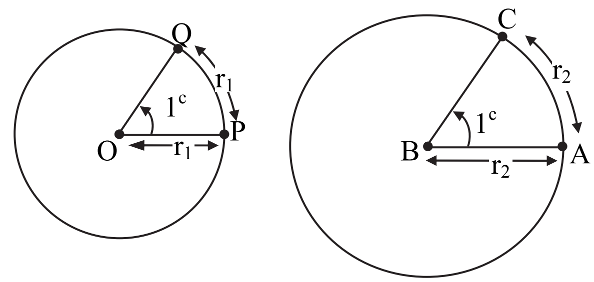


Then by the definition, the measure of ∠POQ is 1 radian (1^c).

Notes:

i. This system of measuring an angle is used in all the higher branches of mathematics.

ii. The radian is a constant angle, therefore radian does not depend on the circle i.e., it does not depend on the radius of the circle as shown below.



In figure we draw two circles of different radii r_1 and r_2 and centres O and B respectively. Then the angle at the centre of both circles is equal to 1^c.

i.e., $\angle POQ = 1^c = \angle ABC$.

Theorem:

A radian is a constant angle. **OR**

Angle subtended at the centre of a circle by an arc whose length is equal to the radius of the circle is always constant.

Proof:

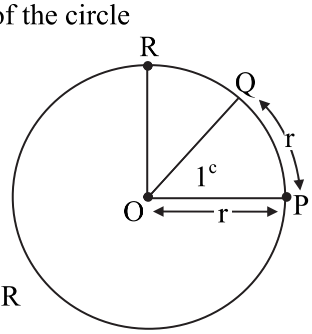
Let O be the centre and r be the radius of the circle. Take points P, Q and R on the circle such that arc PQ = r and ∠POR = 90°.

By definition of radian,

$\angle POQ = 1^c$

arc PR = $\frac{1}{4}\times$ circumference of the circle

$= \frac{1}{4}\times 2\pi r = \frac{\pi r}{2}$



By proportionality theorem

$$\frac{\angle POQ}{\angle POR} = \frac{\text{arc PQ}}{\text{arc PR}}$$

∴ $\angle POQ = \frac{\text{arc PQ}}{\text{arc PR}} \times \angle POR$

∴ $1^c = \frac{r}{\left(\frac{\pi r}{2}\right)} \times 1$ right angle

$= \frac{2}{\pi} \times (1 \text{ right angle})$ ….(i)

i.e., 1^c = constant

∴ R.H.S. of equation (i) is constant and hence a radian is a constant angle.

<u>Relation between degree measure and radian measure:</u>

i. $1^c = \frac{2}{\pi} \times (1.\text{ right angle}) = \frac{2\times 90°}{\pi} = \frac{180°}{\pi}$

ii. $\pi^c = 180°$

iii. $1° = \left(\frac{\pi}{180}\right)^c = 0.01745^c$ (approx.)

$1^c = \left(\frac{180}{\pi}\right)^\circ = \left(\frac{180}{3.142}\right)^\circ = 57° \ 17' \ 48''$

iv. In general $x° = \left(\frac{\pi x}{180}\right)^c$ and $y^c = \left(\frac{180y}{\pi}\right)^\circ$

v. $1^c = \left(\frac{180}{22/7}\right)^\circ = \left(\frac{630}{11}\right)^\circ = \left(57\frac{3}{11}\right)^\circ$

$= 57.3°$ (approx).

Exercise 1.1

1. Determine which of the following pairs of angles are coterminal:

i. 210°, – 150° ii. 330°, – 60°

iii. 405°, – 675° iv. 1230°, – 930°

Solution:

i. $210° - (-150°) = 210° + 150°$
$= 360°$
$= 1(360°)$

which is a multiple of 360°.
Hence, the given angles are coterminal.

ii. $330° - (-60°) = 330° + 60°$
$= 390°$

which is not a multiple of 360°.
Hence, the given angles are not coterminal.

iii. $405° - (-675°) = 405° + 675°$
$= 1080°$
$= 3(360°)$

which is a multiple of 360°.
Hence, the given angles are coterminal.

iv. $1230° - (-930°) = 1230° + 930°$
$= 2160°$
$= 6(360°)$

which is a multiple of 360°.
Hence, the given angles are coterminal.

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2. Express the following angles in degrees:

i. $\left(\frac{5\pi}{12}\right)^c$ **ii.** $\left(-\frac{7\pi}{12}\right)^c$

iii. 8^c **iv.** $\left(\frac{1}{3}\right)^c$

v. $\left(\frac{5\pi}{7}\right)^c$ **vi.** $\left(-\frac{2\pi}{9}\right)^c$

vii. $\left(-\frac{7\pi}{24}\right)^c$

Solution:

i. $\left(\frac{5\pi}{12}\right)^c = \left(\frac{5\pi}{12}\times\frac{180}{\pi}\right)^\circ = 75°$

ii. $\left(-\frac{7\pi}{12}\right)^c = \left(-\frac{7\pi}{12}\times\frac{180}{\pi}\right)^\circ = -105°$

iii. $8^c = \left(8\times\frac{180}{\pi}\right)^\circ = \left(\frac{1440}{\pi}\right)^\circ$

iv. $\left(\frac{1}{3}\right)^c = \left(\frac{1}{3}\times\frac{180}{\pi}\right)^\circ = \left(\frac{60}{\pi}\right)^\circ$

v. $\left(\frac{5\pi}{7}\right)^c = \left(\frac{5\pi}{7} \times \frac{180}{\pi}\right)^\circ = (128.57)^\circ$ approx

vi. $\left(\frac{-2\pi}{9}\right)^c = \left(-\frac{2\pi}{9} \times \frac{180}{\pi}\right)^\circ = -40^\circ$

vii. $\left(\frac{-7\pi}{24}\right)^c = \left(-\frac{7\pi}{24} \times \frac{180}{\pi}\right)^\circ = (-52.5)^\circ$

---

3. Express the following angles in radians:

i. 120° **ii. 225°**

iii. 945° **iv. – 600°**

v. $-\frac{1}{5}^\circ$ **vi. – 108°**

vii. – 144°

Solution:

i. $120^\circ = \left(120 \times \frac{\pi}{180}\right)^c = \left(\frac{2\pi}{3}\right)^c$

ii. $225^\circ = \left(225 \times \frac{\pi}{180}\right)^c = \left(\frac{5\pi}{4}\right)^c$

iii. $945^\circ = \left(945 \times \frac{\pi}{180}\right)^c = \left(\frac{21\pi}{4}\right)^c$

iv. $-600^\circ = \left(-600 \times \frac{\pi}{180}\right)^c = \left(-\frac{10\pi}{3}\right)^c$

v. $\frac{-1}{5}^\circ = \left(-\frac{1}{5} \times \frac{\pi}{180}\right)^c = \left(-\frac{\pi}{900}\right)^c$

vi. $-108^\circ = \left(-108 \times \frac{\pi}{180}\right)^c = \left(-\frac{3\pi}{5}\right)^c$

vii. $-144^\circ = \left(-144 \times \frac{\pi}{180}\right)^c = \left(-\frac{4\pi}{5}\right)^c$

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4. Express the following angles in degrees, minutes and seconds form:

i. (321.9)° **ii. (200.6)°**

Solution:

i. $(321.9)^\circ = 321^\circ + 0.9^\circ$

$= 321^\circ + (0.9 \times 60)'$

$= 321^\circ + 54'$

$= 321^\circ\ 54'$

ii. $(200.6)^\circ = 200^\circ + (0.6)^\circ$

$= 200^\circ + (0.6 \times 60)'$

$= 200^\circ + 36'$

$= 200^\circ\ 36'$

5. If $x^c = 405^\circ$ and $y^\circ = -\frac{\pi}{12}^c$, find x and y.

Solution:

$x^c = 405^\circ$ ……(given)

$\therefore \quad x^c = \left(405 \times \frac{\pi}{180}\right)^c = \left(\frac{9\pi}{4}\right)^c$

$\therefore \quad x = \frac{9\pi}{4}$

Also, $y^\circ = -\frac{\pi}{12}^c$

$\therefore \quad y^\circ = \left(-\frac{\pi}{12} \times \frac{180}{\pi}\right)^\circ = -15^\circ$

$\therefore \quad y = -15$

---

6. If $\theta^\circ = -\frac{5\pi^c}{9}$ and $\phi^c = 900^\circ$, find θ and ϕ.

Solution:

$\theta^\circ = -\frac{5\pi^c}{9}$ …..(given)

$\therefore \quad \theta^\circ = \left(-\frac{5\pi}{9} \times \frac{180}{\pi}\right)^\circ$

$\therefore \quad \theta^\circ = -100^\circ$

$\therefore \quad \theta = -100$

Also, $\phi^c = 900^\circ$

$\therefore \quad \phi^c = \left(900 \times \frac{\pi}{180}\right)^c$

$\therefore \quad \phi^c = 5\pi^c$

$\therefore \quad \phi = 5\pi$

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7. In Δ ABC, $m\angle A = \frac{2\pi^c}{3}$ and $m\angle B = 45^\circ$. Find $m\angle C$ in both the systems.

Solution:

In Δ ABC,

$m\angle A = \frac{2\pi^c}{3} = \left(\frac{2\pi}{3} \times \frac{180}{\pi}\right)^\circ = 120^\circ$

and $m\angle B = 45^\circ$

But, $m\angle A + m\angle B + m\angle C = 180^\circ$

....(sum of measures of angles of a triangle is 180°)

$\therefore \quad 120^\circ + 45^\circ + m\angle C = 180^\circ$

$\therefore \quad m\angle C = 180^\circ - 165^\circ = 15^\circ$

$= \left(15 \times \frac{\pi}{180}\right)^c = \left(\frac{\pi}{12}\right)^c$

$\therefore \quad m\angle C = 15^\circ = \left(\frac{\pi}{12}\right)^c$

8. If the radian measures of two angles of a triangle are as given below. Find the radian measure and the degree measure of the third angle:

i. $\frac{5\pi}{9}, \frac{5\pi}{18}$ **ii.** $\frac{3\pi}{5}, \frac{4\pi}{15}$

Solution:

i. The measures of two angles of a triangle are $\frac{5\pi^c}{9}, \frac{5\pi^c}{18}$

i.e., $\left(\frac{5\pi}{9}\times\frac{180}{\pi}\right)^\circ, \left(\frac{5\pi}{18}\times\frac{180}{\pi}\right)^\circ$

i.e., $100°, 50°$

Let the measure of third angle of the triangle be $x°$.

$\therefore$ $100° + 50° + x° = 180°$

.... (sum of measures of angles of a triangle is 180°)

$\therefore$ $150° + x° = 180°$

$\therefore$ $x° = 180° - 150°$

$\therefore$ $x° = 30°$

$$= \left(30\times\frac{\pi}{180}\right)^c = \frac{\pi}{6}^c$$

$\therefore$ Measure of third angle of the triangle is 30° or $\frac{\pi}{6}^c$.

ii. The measures of two angles of a triangle are $\frac{3\pi^c}{5}, \frac{4\pi^c}{15}$

i.e., $\left(\frac{3\pi}{5}\times\frac{180}{\pi}\right)^\circ, \left(\frac{4\pi}{15}\times\frac{180}{\pi}\right)^\circ$

i.e., $108°, 48°$

Let the measure of third angle of the triangle be $x°$.

$\therefore$ $108° + 48° + x° = 180°$

…. (sum of measures of angles of a triangle is 180°)

$\therefore$ $156° + x° = 180°$

$\therefore$ $x° = 180° - 156°$

$\therefore$ $x° = 24°$

$$= \left(24\times\frac{\pi}{180}\right)^c = \frac{2\pi^c}{15}$$

$\therefore$ Measure of third angle of the triangle is 24° or $\frac{2\pi^c}{15}$.

9. The difference between two acute angles of a right angled triangle is $\frac{3\pi^c}{10}$. Find the angles in degrees.

Solution:

Let the two acute angles measured in degrees be x and y.

$\therefore$ $x + y = 90°$ …..(i)

and $x - y = \left(\frac{3\pi}{10}\right)^c$ …. (given)

$= \left(\frac{3\pi}{10}\times\frac{180}{\pi}\right)^\circ$

$\therefore$ $x - y = 54°$ …..(ii)

Adding (i) and (ii), we get

$2x = 144°$

$\therefore$ $x = 72°$

Putting the value of x in (i), we get

$72° + y = 90°$

$\therefore$ $y = 18°$

Hence, the two acute angles are 72° and 18°.

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10. The sum of two angles is $5\pi^c$ and their difference is 60°. Find the angles in degrees.

Solution:

Let the two acute angles measured in degrees be x and y.

$\therefore$ $x + y = 5\pi^c$ ….. (given)

$\therefore$ $x + y = \left(5\pi\times\frac{180}{\pi}\right)^\circ$

$\therefore$ $x + y = 900°$ …..(i)

and $x - y = 60°$ …..(ii) (given)

Adding (i) and (ii), we get

$2x = 960°$

$\therefore$ $x = 480°$

Putting the value of x in (i), we get

$480° + y = 900°$

$\therefore$ $y = 420°$

Hence, the two angles are 480° and 420°.

---

11. The measures of angles of a triangle are in the ratio 2: 3: 5. Find their measures in radians.

Solution:

Let the measures of angles of the triangle be 2k, 3k, 5k in degrees.

$\therefore$ $2k + 3k + 5k = 180°$

….(sum of measures of angles of a triangle is 180°)

$\therefore$ 10k = 180°

$\therefore$ k = 18°

$\therefore$ the measures of three angles are

2k = 2 × 18° = 36°

3k = 3 × 18° = 54°

5k = 5 × 18° = 90°

These three angles in radians are

$$36° = \left(36 \times \frac{\pi}{180}\right)^c = \frac{\pi}{5}^c$$

$$54° = \left(54 \times \frac{\pi}{180}\right)^c = \frac{3\pi}{10}^c$$

$$90° = \left(90 \times \frac{\pi}{180}\right)^c = \frac{\pi}{2}^c.$$

12. One angle of a triangle has measure $\frac{2\pi^c}{9}$ and the measures of other two angles are in the ratio 4 : 3, find their measures in degrees and radians.

Solution:

$$\frac{2\pi^c}{9} = \left(\frac{2\pi}{9} \times \frac{180}{\pi}\right)^\circ = 40°$$

Let the measures of other two angles of the triangle be 4k and 3k in degrees.

$\therefore$ 40° + 4k + 3k = 180°

$\therefore$ 7k = 140°

$\therefore$ k = 20°

$\therefore$ the measures of two angles are

4k = 4 × 20° = 80°

3k = 3 × 20° = 60°

These two angles in radians are

$$80° = \left(80 \times \frac{\pi}{180}\right)^c = \frac{4\pi}{9}^c$$

$$60° = \left(60 \times \frac{\pi}{180}\right)^c = \frac{\pi}{3}^c$$

13. The measures of angles of a quadrilateral are in the ratio 3 : 4 : 5 : 6. Find their measures in radians.

Solution:

Let the measures of angles of the quadrilateral be

3k, 4k, 5k, 6k in degrees.

$\therefore$ 3k + 4k + 5k + 6k = 360°

….(sum of measures of angles of a quadrilateral is 360°)

$\therefore$ 18k = 360°

$\therefore$ k = 20°

$\therefore$ the measures of angles are

3k = 3 × 20° = 60°

4k = 4 × 20° = 80°

5k = 5 × 20° = 100°

6k = 6 × 20° = 120°

These angles in radians are

$$60° = \left(60 \times \frac{\pi}{180}\right)^c = \frac{\pi^c}{3}$$

$$80° = \left(80 \times \frac{\pi}{180}\right)^c = \frac{4\pi^c}{9}$$

$$100° = \left(100 \times \frac{\pi}{180}\right)^c = \frac{5\pi^c}{9}$$

$$120° = \left(120 \times \frac{\pi}{180}\right)^c = \frac{2\pi^c}{3}$$

14. One angle of a quadrilateral has measure $\frac{2\pi^c}{5}$ and the measures of other three angles are in the ratio 2 : 3 : 4. Find their measures in radians and in degrees.

Solution:

$$\frac{2\pi^c}{5} = \left(\frac{2\pi}{5} \times \frac{180}{\pi}\right)^\circ = 72°$$

Let the measures of other three angles of the quadrilateral be 2k, 3k, 4k in degrees.

$\therefore$ 72° + 2k + 3k + 4k = 360°

$\therefore$ 9k = 288°

$\therefore$ k = 32°

$\therefore$ the measures of angles are

2k = 2 × 32° = 64°

3k = 3 × 32° = 96°

4k = 4 × 32° = 128°

$\therefore$ the angles of the quadrilateral in degrees are 72°, 64°, 96°, 128°.

The angles in radians are

$$64° = \left(64 \times \frac{\pi}{180}\right)^c = \frac{16\pi^c}{45}$$

$$96° = \left(96 \times \frac{\pi}{180}\right)^c = \frac{8\pi^c}{15}$$

$$128° = \left(128 \times \frac{\pi}{180}\right)^c = \frac{32\pi^c}{45}$$

Length of an arc and area of sector of a circle

Theorem:

If S is the length of an arc of a circle of radius r which subtends an angle θ^c at the centre of the circle, then S = rθ.

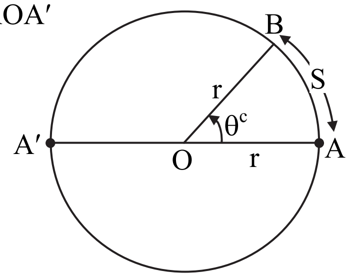
Proof:

Let O be the centre and r be the radius of the circle.
Let AB be an arc of the circle with length 'S' units and m∠AOB = θ^c.
Let AA′ be the diameter of the circle

(Note that θ is measured in radians)

Now, ℓ(arc AB) ∝ m ∠AOB

and ℓ(arc ABA′) ∝ m ∠AOA′



$\therefore \quad \dfrac{\ell(\text{arc} AB)}{\ell(\text{arc } ABA')} = \dfrac{\theta^c}{\pi}$

$\therefore \quad \dfrac{S}{\frac{1}{2}(\text{circumference})} = \dfrac{\theta}{\pi}$

$\therefore \quad \dfrac{S}{\pi r} = \dfrac{\theta}{\pi}$

∴ S = rθ

∴ Length of an arc, S = rθ.

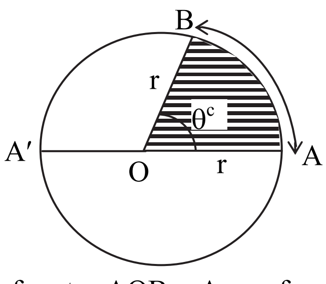
***Theorem*:**

If θ^c is an angle between two radii of the circle of radius r, then the area of the corresponding sector is $\frac{1}{2}r^2\theta$.

Proof:

Let O be the centre and r be the radius of the circle and m ∠AOB = θ^c.
Let AA′ be the diameter of the circle
Area of sector AOB ∝ m ∠AOB
and area of sector ABA′ ∝ m ∠AOA′

$\therefore \quad \dfrac{\text{Area of sector AOB}}{\text{Area of sector ABA}'} = \dfrac{m\angle AOB}{m\angle AOA'} = \dfrac{\theta}{\pi}$



$\therefore \quad \text{Area of sector AOB} = \text{Area of sector ABA}' \times \dfrac{\theta}{\pi}$

$= \dfrac{1}{2}(\pi r^2) \times \dfrac{\theta}{\pi} = \dfrac{1}{2}r^2\theta$

$\therefore \quad \text{Area of sector AOB} = \dfrac{1}{2}r^2\theta.$

Note:

$\text{A(sector)} = \dfrac{1}{2} \times r^2 \times \theta = \dfrac{1}{2} \times r \times r\theta$

$= \dfrac{1}{2} \times r \times S$

Note:

The above theorems are not asked in examination but are provided just for reference.

Exercise 1.2

1. Find the length of an arc of a circle which subtends an angle of 108° at the centre, if the radius of the circle is 15 cm.

Solution:

Here, r = 15cm and

$\theta = 108° = \left(108 \times \dfrac{\pi}{180}\right)^c = \dfrac{3\pi}{5}^c$

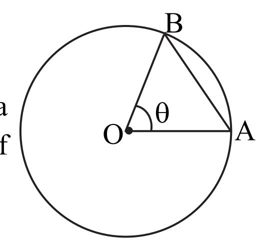
Since, S = r.θ

$\therefore \quad S = 15 \times \dfrac{3\pi}{5} = 9\pi$ cm.

---

2. The radius of a circle is 9 cm. Find the length of an arc of this circle which cuts off a chord of length equal to length of radius.

Solution:



Here, r = 9cm

Let the arc AB cut off a chord equal to the radius of the circle.

∴ Δ OAB is an equilateral triangle.

∴ *m*∠AOB = 60°

$\therefore \quad \theta = 60° = \left(60 \times \dfrac{\pi}{180}\right)^c = \dfrac{\pi}{3}^c$

Since, S = r.θ

$\therefore \quad S = 9 \times \dfrac{\pi}{3} = 3\pi$ cm.

3. Find in radians and degrees the angle subtended at the centre of a circle by an arc whose length is 15 cm, if the radius of the circle is 25 cm.

Solution:

Here, r = 25 cm and S = 15 cm

Since, S = r.θ

∴ $15 = 25 \times \theta$

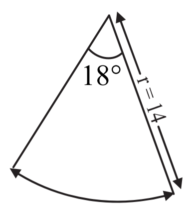
∴ $\theta = \left(\frac{15}{25}\right)^c$

∴ $\theta = \left(\frac{3}{5}\right)^c = \left(\frac{3}{5} \times \frac{180}{\pi}\right)^\circ = \frac{108}{\pi}^\circ$

---

4. A pendulum of 14 cm long oscillates through an angle of 18°. Find the length of the path described by its extremity.

Solution:



Here, r = 14 cm and

$\theta = 18^\circ = \left(18 \times \frac{\pi}{180}\right)^c = \frac{\pi}{10}^c$

Since, $S = r.\theta = 14 \times \frac{\pi}{10}$

∴ $S = \frac{7\pi}{5}$ cm.

---

5. Two arcs of the same length subtend angles of 60° and 75° at the centres of the circles. What is the ratio of radii of two circles?

Solution:

Let r_1 and r_2 be the radii of the given circles and let their arcs of same length S subtend angles of 60° and 75° at their centres.

Angle subtended at the centre of the first circle,

$\theta_1 = 60^\circ = \left(60 \times \frac{\pi}{180}\right)^c = \frac{\pi}{3}^c$

∴ $S = r_1\theta_1 = r_1\left(\frac{\pi}{3}\right)$ ….(i)

Angle subtended at the centre of the second circle,

$\theta_2 = 75^\circ = \left(75 \times \frac{\pi}{180}\right)^c = \frac{5\pi}{12}^c$

∴ $S = r_2\theta_2 = r_2\left(\frac{5\pi}{12}\right)$ ….(ii)

From (i) and (ii), we get

$r_1\left(\frac{\pi}{3}\right) = r_2\left(\frac{5\pi}{12}\right)$

∴ $\frac{r_1}{r_2} = \frac{15}{12}$

∴ $\frac{r_1}{r_2} = \frac{5}{4}$

∴ $r_1 : r_2 = 5 : 4$.

---

6. The area of the circle is 25π sq.cm. Find the length of its arc subtending an angle of 144° at the centre. Also find the area of the corresponding sector.

Solution:

Area of circle = πr^2

But area is given to be 25 π sq.cm

∴ $25\pi = \pi r^2$

∴ $r^2 = 25$ ∴ r = 5 cm

$\theta = 144^\circ = \left(144 \times \frac{\pi}{180}\right)^c = \frac{4\pi}{5}^c$

Since, $S = r\theta = 5\left(\frac{4\pi}{5}\right) = 4\pi$ cm.

A(sector) $= \frac{1}{2} \times r \times S = \frac{1}{2} \times 5 \times 4\pi = 10\pi$ sq.cm.

---

7. OAB is a sector of the circle with centre O and radius 12 cm. If *m*∠AOB = 60°, find the difference between the areas of sector AOB and ΔAOB.

Solution:

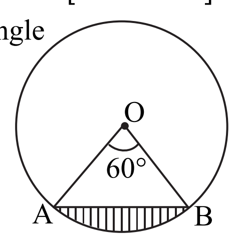
Here, OA = OB = r = 12 cm

Given *m*∠AOB = 60°

m∠OAB = m∠OBA …[∵ OA = OB]

∴ ΔOAB is an equilateral triangle

∴ $\theta = 60^\circ = \left(60 \times \frac{\pi}{180}\right)^c = \frac{\pi}{3}^c$



Now,

A (sector AOB) – A(ΔAOB) $= \frac{1}{2} r^2\theta - \frac{\sqrt{3}}{4}.(\text{side})^2$

$= \frac{1}{2} \times (12)^2 \times \frac{\pi}{3} - \frac{\sqrt{3}}{4}.(12)^2$

$= \frac{1}{2}\times 144\times\frac{\pi}{3} - \frac{\sqrt{3}}{4}.(144)$

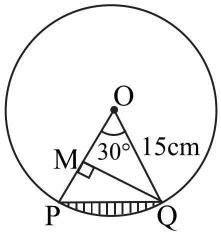
$= 24\pi - 36\sqrt{3}$

$= 12(2\pi - 3\sqrt{3})$ sq.cm.

8. OPQ is a sector of a circle with centre O and radius 15 cm. If $m\angle POQ = 30°$, find the area enclosed by arc PQ and chord PQ.

Solution:

Here, r = 15 cm



$m\angle POQ = 30° = \left(30\times\frac{\pi}{180}\right)^c$

$\therefore\quad \theta = \frac{\pi}{6}^c$

Draw QM ⊥ OP

$\therefore\quad \sin 30° = \frac{QM}{15}$

$\therefore\quad QM = 15 \times \frac{1}{2} = \frac{15}{2}$

Shaded portion indicates the area enclosed by arc PQ and chord PQ.

$\therefore$ A(shaded portion) = A(sector OPQ) − A(ΔOPQ)

$= \frac{1}{2}r^2\theta - \frac{1}{2}\times OP\times QM$

$= \frac{1}{2}\times 15^2\times\frac{\pi}{6} - \frac{1}{2}\times 15\times\frac{15}{2}$

$= \frac{75\pi}{4} - \frac{225}{4}$

$= \frac{75}{4}(\pi - 3)$ sq.cm.

9. The perimeter of a sector of a circle, of area 25π sq.cm, is 20 cm. Find the area of sector.

Solution:

Area of circle = πr^2

But area is given to be 25π sq.cm

$\therefore\quad 25\pi = \pi r^2$

$\therefore\quad r = 5$ cm

Perimeter of sector = 2r + S

But perimeter is given to be 20 cm

$\therefore\quad 20 = 10 + S$

$\therefore\quad S = 10$ cm

Area of sector $= \frac{1}{2}\times r\times S = \frac{1}{2}\times 5\times 10$

$= 25$ sq.cm.

10. The perimeter of a sector of a circle, of area 64π sq.cm, is 56 cm. Find the area of sector.

Solution:

Area of circle = πr^2

But area is given to be 64 π sq.cm

$\therefore\quad 64\pi = \pi r^2$

$\therefore\quad r = 8$ cm

Perimeter of sector = 2r + S

But perimeter is given to be 56 cm

$\therefore\quad 56 = 16 + S$

$\therefore\quad S = 40$ cm

Area of sector $= \frac{1}{2}\times r\times S = \frac{1}{2}\times 8\times 40$

$= 160$ sq.cm.

Miscellaneous Exercise - 1

1. Express the following angles into radians:

i. 50° 37′ 30″ ii. – 10° 40′ 30″

Solution:

i. $50° 37' 30'' = \left(50 + \frac{37}{60} + \frac{30}{60\times 60}\right)^\circ$

$= (50 + 0.6166 + 0.00833)°$

$= 50.625° = \left(50.625\times\frac{\pi}{180}\right)^c$

ii. $-10° 40' 30'' = -\left(10 + \frac{40}{60} + \frac{30}{60\times 60}\right)^\circ$

$= -(10 + 0.66 + 0.0083)°$

$= -(10.675)° = -\left(10.675\times\frac{\pi}{180}\right)^c$

2. Express the following angles in degrees, minutes and seconds.

i. (11.0133)° ii. (94.3366)°

Solution:

i. (11.0133)° = 11° + (0.0133)°
= 11° + (0.0133 × 60)′
= 11° + (0.798)′
= 11° + (0.798 × 60)″
= 11° + (47.88)″
= 11° 48″ (approx)

ii. (94.3366)° = 94° + (0.3366)°
= 94° + (0.3366 × 60)′
= 94° + (20.196)′
= 94° + 20′ + (0.196)′
= 94° + 20′ + (0.196 × 60)″
= 94° + 20′ + (11.76)″
= 94° 20′ 12″ (approx)

3. In Δ LMN, $m\angle L = \frac{3\pi^c}{4}$ and $m\angle N = 30°$. Find the measure of ∠M both in degrees and radians.

Solution:

In ΔLMN,

$m\angle L = \frac{3\pi}{4}^c = \left(\frac{3\pi}{4} \times \frac{180}{\pi}\right)^{\circ} = 135°$

and $m\angle N = 30°$

But $m\angle L + m\angle M + m\angle N = 180°$

∴ $135° + m\angle M + 30° = 180°$

∴ $m\angle M = 180° - 165° = 15°$

$= \left(15 \times \frac{\pi}{180}\right)^c$

∴ $m\angle M = \frac{\pi}{12}^c$

∴ Measure of $\angle M = 15° = \frac{\pi}{12}^c$

4. Find the radian measure of the interior angle of a regular

i. Pentagon ii. Hexagon

iii. Octagon.

Solution:

i. Pentagon:

Number of sides = 5

Number of exterior angles = 5

Sum of exterior angles = 360°

∴ Each exterior angle $= \frac{360°}{\text{no. of sides}} = \frac{360°}{5}$

∴ Each interior angle $= \left(180 - \frac{360}{5}\right)^{\circ}$

$= (180 - 72)°$

$= 108° = \left(108 \times \frac{\pi}{180}\right)^c$

$= \frac{3\pi^c}{5}$

ii. Hexagon:

Number of sides = 6

Number of exterior angles = 6

Sum of exterior angles = 360°

∴ Each exterior angle $= \frac{360°}{\text{no. of sides}} = \frac{360°}{6}$

∴ Each interior angle $= \left(180 - \frac{360}{6}\right)^{\circ}$

$= (180 - 60)°$

$= 120° = \left(120 \times \frac{\pi}{180}\right)^c$

$= \frac{2\pi^c}{3}$

iii. Octagon:

Number of sides = 8

Number of exterior angles = 8

Sum of exterior angles = 360°

∴ Each exterior angle $= \frac{360°}{\text{no. of sides}} = \frac{360°}{8}$

∴ Each interior angle $= \left(180 - \frac{360}{8}\right)^{\circ}$

$= (180 - 45)°$

$= 135° = \left(135 \times \frac{\pi}{180}\right)^c$

$= \frac{3\pi^c}{4}$

5. Find the number of sides of a regular polygon, if each of its interior angles is $\frac{3\pi^c}{4}$.

Solution:

Each interior angle of a regular polygon $= \frac{3\pi}{4}^c$

$= \left(\frac{3\pi}{4} \times \frac{180}{\pi}\right)^{\circ}$

$= 135°$

∴ Exterior angle = 180° – 135° = 45°.

Let the number of sides of the regular polygon be n.

But in a regular polygon,

exterior angle $= \frac{360°}{\text{no. of sides}}$

∴ $45° = \frac{360°}{n}$

∴ $n = \frac{360°}{45°} = 8$

∴ Number of sides of a regular polygon = 8.

6. Two circles each of radius 7 cm, intersect each other. The distance between their centres is $7\sqrt{2}$ cm. Find the area common to both the circles.

Solution:

Let O and O_1 be the centres of two circles intersecting each other at A and B.

Then, $OA = OB = O_1A = O_1B = 7$ cm

and $OO_1 = 7\sqrt{2}$ cm

$\therefore \quad OO_1^2 = 98$ …..(i)

Since, $OA^2 + O_1A^2 = 7^2 + 7^2$

$= 98$

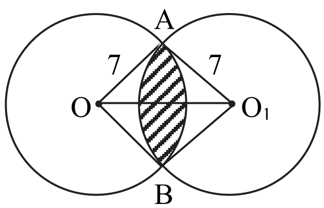
$= OO_1^2$ ….[From (i)]

$\therefore \quad m\angle OAO_1 = 90°$

$\therefore \quad \square OAO_1B$ is a square.

$m\angle AOB = m\angle AO_1B = 90°$

$$= \left(90 \times \frac{\pi}{180}\right)^c = \frac{\pi}{2}^c$$



Now, $A(\text{sector } OAB) = \frac{1}{2}r^2\theta$

$= \frac{1}{2} \times 7^2 \times \frac{\pi}{2}$

$= \frac{49\pi}{4}$ sq.cm

and $A(\text{sector } O_1AB) = \frac{1}{2}r^2\theta$

$= \frac{1}{2} \times 7^2 \times \frac{\pi}{2}$

$= \frac{49\pi}{4}$ sq.cm

$A(\square OAO_1B) = (\text{side})^2 = 49$ sq.cm

$\therefore$ required area = area of shaded portion

$= A(\text{sector } OAB) + A(\text{sector } O_1AB) - A(\square OAO_1B)$

$= \frac{49\pi}{4} + \frac{49\pi}{4} - 49$

$= \frac{49\pi}{2} - 49$

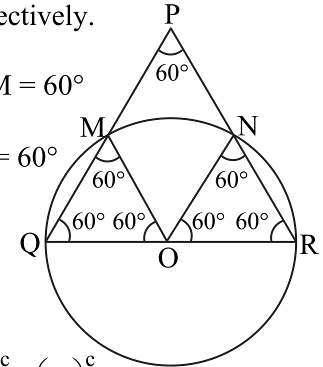
$= \frac{49}{2}(\pi - 2)$ sq.cm

7. ΔPQR is an equilateral triangle with side 18 cm. A circle is drawn on segment QR as diameter. Find the length of the arc of this circle intercepted within the triangle.

Solution:

Let 'O' be the centre of the circle drawn on QR as a diameter.

Let the circle intersects seg PQ and PR at points M and N respectively.



Since, $l(OQ) = l(OM)$

$\therefore \quad m\angle OMQ = m\angle OQM = 60°$

$\therefore \quad m\angle MOQ = 60°$

Similarly, $m\angle NOR = 60°$

QR = 18 cm

$\therefore$ r = 9 cm

$$\theta = 60° = \left(60 \times \frac{\pi}{180}\right)^c = \left(\frac{\pi}{3}\right)^c$$

$\therefore \quad l(\text{arc } MN) = S = r\theta = 9 \times \frac{\pi}{3} = 3\pi$ cm.

8. Find the radius of the circle in which a central angle of 60° intercepts an arc of length 37.4 cm.

$\left(\text{use } \pi = \frac{22}{7}\right)$

Solution:

Let S be the length of the arc and r be the radius of the circle.

$$\theta = 60° = \left(60 \times \frac{\pi}{180}\right)^c = \frac{\pi}{3}^c$$

S = 37.4 cm

Since, $S = r\theta$

$\therefore \quad 37.4 = r \times \frac{\pi}{3}$

$\therefore \quad 3 \times 37.4 = r \times \frac{22}{7}$

$\therefore \quad r = \frac{3 \times 37.4 \times 7}{22}$

$\therefore \quad r = 35.7$ cm

9. A wire of length 10 cm is bent so as to form an arc of a circle of radius 4 cm. What is the angle subtended at the centre in degrees?

Solution:

S = 10 cm and

r = 4 cm ….(given)

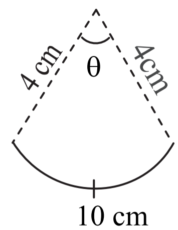
Since, $S = r\theta$

$\therefore \quad 10 = 4 \times \theta$

$\therefore \quad \theta = \left(\frac{5}{2}\right)^c$

$= \left(\frac{5}{2} \times \frac{180}{\pi}\right)^\circ$

$= \left(\frac{450}{\pi}\right)^\circ$



10. If the arcs of the same lengths in two circles subtend angles 65° and 110° at the centre. Find the ratio of their radii.

Solution:

Let r_1 and r_2 be the radii of the given circles and let their arcs of same length S subtend angles of 65° and 110° at their centres.

Angle subtended at the centre of the first circle,

$\theta_1 = 65^\circ = \left(65 \times \frac{\pi}{180}\right)^c = \frac{13\pi}{36}^c$

$\therefore \quad S = r_1\theta_1 = r_1\left(\frac{13\pi}{36}\right)$ ....(i)

Angle subtended at the centre of the second circle,

$\theta_2 = 110^\circ = \left(110 \times \frac{\pi}{180}\right)^c = \frac{11\pi}{18}^c$

$\therefore \quad S = r_2\theta_2 = r_2\left(\frac{11\pi}{18}\right)$ ....(ii)

From (i) and (ii), we get

$r_1\left(\frac{13\pi}{36}\right) = r_2\left(\frac{11\pi}{18}\right)$

$\therefore \quad \frac{r_1}{r_2} = \frac{22}{13}$

$\therefore \quad r_1 : r_2 = 22 : 13$

11. Find the area of a sector whose arc length is 30π cm and the angle of the sector is 40°.

Solution:

Let S be the length of the arc.

$\theta = 40^\circ = \left(40 \times \frac{\pi}{180}\right)^c = \frac{2\pi}{9}^c$

and $S = 30\pi$ cm

Since, $S = r\theta$

$\therefore \quad 30\pi = r \times \frac{2\pi}{9}$

$\therefore \quad r = \frac{30\pi \times 9}{2\pi}$ $\quad \therefore \quad r = 135$ cm

Area of sector $= \frac{1}{2} \times r \times S$

$= \frac{1}{2} \times 135 \times 30\pi$

$= 2025 \times \frac{22}{7}$

$= 6364.28$ sq.cm

12. The area of a circle is 81π sq.cm. Find the length of the arc subtending an angle of 300° at the centre and the area of corresponding sector.

Solution:

Area of circle $= \pi r^2$

But area is given to be 81π sq.cm

$\therefore \quad \pi r^2 = 81\pi$

$\therefore \quad r^2 = 81$

$\therefore \quad r = 9$ cm

$\theta = 300^\circ = \left(300 \times \frac{\pi}{180}\right)^c = \frac{5\pi}{3}^c$

Since, $S = r\theta = 9 \times \frac{5\pi}{3} = 15\pi$ cm.

Area of sector $= \frac{1}{2} \times r \times S$

$= \frac{1}{2} \times 9 \times 15\pi$

$= \frac{135\pi}{2}$ sq.cm

13. The measures of angles of a quadrilateral are in the ratio 2 : 3 : 6 : 7. Find their measures in degrees and in radians.

Solution:

Let the measures of angles of the quadrilateral be 2k, 3k, 6k and 7k in degrees.

$\therefore \quad 2k + 3k + 6k + 7k = 360^\circ$

....(sum of measures of angles of a quadrilateral is 360°)

$\therefore \quad 18k = 360^\circ$

$\therefore \quad k = 20^\circ$

$\therefore$ the measures of angles are

$2k = 2 \times 20^\circ = 40^\circ$

$3k = 3 \times 20^\circ = 60^\circ$

$6k = 6 \times 20^\circ = 120^\circ$

$7k = 7 \times 20^\circ = 140^\circ$

These angles in radians are

$40^\circ = \left(40 \times \frac{\pi}{180}\right)^c = \frac{2\pi^c}{9}$

$$60^\circ = \left(60 \times \frac{\pi}{180}\right)^c = \frac{\pi^c}{3}$$

$$120^\circ = \left(120 \times \frac{\pi}{180}\right)^c = \frac{2\pi^c}{3}$$

$$140^\circ = \left(140 \times \frac{\pi}{180}\right)^c = \frac{7\pi^c}{9}$$

14. The angles of a triangle are in A.P. and the greatest angle is 84°. Find all the three angles in radians.

Solution:

Let the measures of angles of a triangle be a – d, a, a + d in degrees.

∴ (a – d) + a + (a + d) = 180°

….(sum of measures of angles of a triangle is 180°)

∴ 3a = 180°

∴ a = 60°

But a + d = 84° …..[greatest angle is 84°]

∴ 60° + d = 84°

∴ d = 24°

∴ the measures of angles are

a – d = 60° – 24° = 36°

a = 60°

and a + d = 60° + 24° = 84°

These angles in radians are

$$36^\circ = \left(36 \times \frac{\pi}{180}\right)^c = \frac{\pi^c}{5}$$

$$60^\circ = \left(60 \times \frac{\pi}{180}\right)^c = \frac{\pi^c}{3}$$

$$84^\circ = \left(84 \times \frac{\pi}{180}\right)^c = \frac{7\pi^c}{15}$$

15. Show that the minute-hand of a clock gains 5° 30′ on the hour-hand in one minute.

Solution:

Angle made by hour-hand in one minute

$$= \frac{360^\circ}{12 \times 60} = \left(\frac{1}{2}\right)^\circ$$

Angle made by minute-hand in one minute

$$= \frac{360^\circ}{60} = 6^\circ$$

∴ Gain by minute-hand on the hour-hand in one minute

$$= 6^\circ - \left(\frac{1}{2}\right)^\circ = 5^\circ + \left(\frac{1}{2}\right)^\circ = 5^\circ\ 30'$$

16. Find the angle between the hour-hand and minute-hand of a clock at

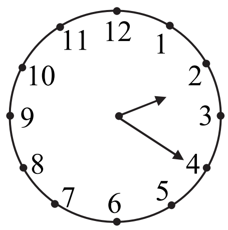
i. twenty minutes past two

ii. quarter past six

iii. ten past eleven.

Solution:

i. At 2 : 20, the minute-hand is at mark 4 and hour hand has crossed $\left(\frac{1}{3}\right)^{rd}$ of angle between 2 and 3.



Angle between two consecutive marks

$$= \frac{360^\circ}{12} = 30^\circ$$

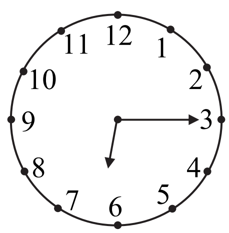
Angle traced by hour-hand in 20 minutes

$$= \frac{1}{3}(30^\circ) = 10^\circ$$

Angle between marks 2 and 4 = 2 × 30° = 60°

∴ Angle between two hands of the clock at twenty minutes past two = 60° – 10° = 50°

ii. At 6:15, the minute-hand is at mark 3 and hour hand has crossed $\frac{1}{4}^{th}$ of the angle between 6 and 7.



Angle between two consecutive marks

$$= \frac{360^\circ}{12} = 30^\circ$$

Angle traced by hour-hand in 15 minutes

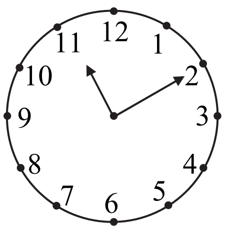
$$= \frac{1}{4}(30^\circ) = (7.5)^\circ = \left(7\frac{1}{2}\right)^\circ$$

Angle between mark 3 and 6 = 3 × 30° = 90°

∴ Angle between two hands of the clock at quarter past six = $90^\circ + 7\frac{1}{2}^\circ$

$$= \left(97\frac{1}{2}\right)^\circ$$

iii. At 11:10, the minute-hand is at mark 2 and hour hand has crossed $\frac{1}{6}^{\text{th}}$ of the angle between 11 and 12



Angle between two consecutive marks

$= \frac{360^\circ}{12} = 30^\circ$

Angle traced by hour hand in 10 minutes

$= \frac{1}{6}(30^\circ) = 5^\circ$

Angle between mark 11 and 2 = 3 × 30° = 90°

∴ Angle between two hands of the clock at ten past eleven = 90° – 5° = 85°

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17. A train is running on a circular track of radius 1 km at the rate of 36 km per hour. Find the angle to the nearest minute, through which it will turn in 30 seconds.

Solution:

r = 1km = 1000m

ℓ(Arc covered by train in 30 seconds)

$= 30 \times \frac{36000}{60 \times 60}$ m

∴ S = 300 m

Since, S = rθ

∴ 300 = 1000 × θ

∴ $\theta = \left(\frac{3}{10}\right)^c$

$= \left(\frac{3}{10} \times \frac{180}{\pi}\right)^\circ$

$= \left(\frac{54}{\pi}\right)^\circ$

$= \left(\frac{54 \times 7}{22}\right)^\circ$

= (17.18)°

= 17° + 0.18°

= 17° + (0.18 × 60)′

= 17° + (10.8)′

∴ θ = 17° 11′ (approx)

18. The angles of a triangle are in A.P. and the ratio of the number of degrees in the least to the number of radians in the greatest is 60 : π. Find the angles of the triangle in degrees and radians.

Solution:

Let the measures of angles of a triangle be a – d, a, a + d in degrees.

∴ (a – d) + a + (a + d) = 180°

….(sum of measures of angles of a triangle is 180°)

∴ 3a = 180°

∴ a = 60°

Also, greatest angle in radians $= (a + d) \times \frac{\pi}{180}^c$

According to the given condition,

$$\frac{a - d}{(a + d) \times \frac{\pi}{180}} = \frac{60}{\pi}$$

∴ $\frac{(60 - d)180}{60 + d} = 60$

∴ $\frac{(60 - d)3}{60 + d} = 1$

…. [Dividing throughout by 60°]

∴ 180 – 3d = 60 + d

∴ 120 = 4d

∴ d = 30°

∴ the measures of angles are

a – d = 60° – 30° = 30°

a = 60°

and a + d = 60° + 30° = 90°

These angles in radians are

$$30^\circ = \left(30 \times \frac{\pi}{180}\right)^c = \frac{\pi^c}{6}$$

$$60^\circ = \left(60 \times \frac{\pi}{180}\right)^c = \frac{\pi^c}{3}$$

$$90^\circ = \left(90 \times \frac{\pi}{180}\right)^c = \frac{\pi^c}{2}$$

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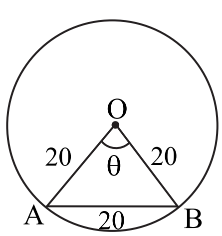
19. In a circle of diameter 40 cm, the length of a chord is 20 cm. Find the length of minor arc of the chord.

Solution:

Let 'O' be the centre of the circle and AB be the chord of the circle.



Here, d = 40 cm

∴ r = 20 cm

The angle subtended at the centre by the minor arc AOB is $\theta = 60° = \left(60 \times \frac{\pi}{180}\right)^c = \frac{\pi}{3}^c$

$\therefore$ ℓ(minor arc of chord AB) $= r\theta = 20 \times \frac{\pi}{3}$

$= \frac{20\pi}{3}$ cm.

20. The angles of a quadrilateral are in A.P. and the greatest angle is double the least. Express the least angle in radians.

Solution:

Let measures of angles of quadrilateral be

a – 3d, a – d, a + d, a + 3d in degrees.

$\therefore$ (a – 3d) + (a – d) + (a + d) + (a + 3d) = 360°

….(sum of measures of angles of a quadrilateral is 360°)

$\therefore$ 4a = 360°

$\therefore$ a = 90°

Also, a + 3d = 2.(a – 3d)

$\therefore$ 90° + 3d = 2.(90° – 3d)

$\therefore$ 90° + 3d = 180° – 6d

$\therefore$ 9d = 90°

$\therefore$ d = 10°

$\therefore$ Measure of least angle = a – 3d = 90° – 3(10°)

= 90° – 30°

= 60°

$= \left(60 \times \frac{\pi}{180}\right)^c$

$= \frac{\pi}{3}^c$

Additional Problems for Practice

Based on Exercise 1.1

1. Determine which of the following pairs of angles are coterminal:
   i. 420°, – 300°
   ii. 330°, –45°

2. Express the following angles in degrees:
   i. $\left(\frac{5\pi}{8}\right)^c$ ii. $\left(-\frac{5\pi}{6}\right)^c$
   iii. 6^c iv. $\left(\frac{1}{4}\right)^c$
   v. $(1.1)^c$

3. Express the following angles in radians:
   i. 150° ii. 340°
   iii. – 225° iv. $-\left(\frac{1}{4}\right)^\circ$

4. Express the following angles in degrees, minutes and seconds form:
   i. (125.3)° ii. (50.9)°
   iii. $\left(\frac{11}{16}\right)^c$

5. If $\theta° = -\frac{2\pi^c}{9}$ and $\phi^c = 450°$, find θ and ϕ.

6. In ΔPQR, m∠P = 40° and m∠Q = $\frac{4\pi^c}{9}$, find the radian measure and the degree measure of ∠R.

7. The difference between two acute angles of a right angled triangle is $\frac{\pi^c}{9}$. Find the angles in degrees.

8. The sum of two angles is $3\pi^c$ and their difference is 40°. Find the angles in degrees.

9. i. The measures of angles of a triangle are in the ratio 2 : 6 : 7. Find their measures in degrees.
   ii. The measures of angles of a quadrilateral are in the ratio 2 : 3 : 5 : 8. Find their measures in radians.

10. One angle of a quadrilateral has measure $\frac{\pi^c}{3}$ and the measures of other three angles are in the ratio 4 : 5 : 6. Find their measures in degrees and in radians.

Based on Exercise 1.2

1. Find the length of the arc of circle of diameter 6 cm, if the arc is subtending an angle of 120° at the centre.

2. Find the length of an arc of a circle which subtends an angle of 144° at the centre, if the radius of the circle is 5 cm.

3. The radius of a circle is 7 cm. Find the length of an arc of this circle which cuts off a chord of length equal to radius.

4. A pendulum 18 cm long oscillates through an angle of 32°. Find the length of the path described by its extremity.

5. Two arcs of the same length subtend angles of 60° and 80° at the centre of the circles. What is the ratio of radii of two circles?

6. Find the area of the sector of circle which subtends an angle of 60° at the centre, if the radius of the circle is 3 cm.

7. The area of the circle is 64π sq. cm. Find the length of its arc subtending an angle of 120° at the centre. Also, find the area of the corresponding sector.

8. If the perimeter of a sector of a circle is four times the radius of the circle, find the central angle of corresponding sector in radians.

9. OPQ is a sector of a circle with centre O and radius 12 cm. If m ∠POQ = 60°, find the area enclosed by arc PQ and chord PQ.

10. The perimeter of a sector of a circle, of area 49π sq. cm, is 44 cm. Find the area of sector.

Based on Miscellaneous Exercise - 1

1. Express the following angles into radians:
   i. 5° 37′ 30″ ii. –35° 40′ 30″

2. Express the following angles in degrees, minutes and seconds:
   i. (83.1161)° ii. (17.0127)°

3. A wheel makes 360 revolutions in 1 minute. Through how many radians does it turn in 1 second?

4. In ΔXYZ, m∠X = $\frac{5\pi^c}{12}$ and m∠Z = 60°. Find the measure of ∠Y both in degrees and radians.

5. Find the radian measure of the interior angle of a regular heptagon.

6. Find the number of sides of a regular polygon, if each of its interior angles is $\frac{2\pi^c}{3}$.

7. Two circles each of radius 5 cm intersect each other. The distance between their centres is $5\sqrt{2}$ cm. Find the area common to both the circles.

8. ΔABC is an equilateral triangle with side 6 cm. A circle is drawn on segment BC as diameter. Find the length of the arc of this circle intercepted within ΔABC.

9. Find the radius of the circle in which a central angle of 60° intercepts an arc of length 28.6 cm. $\left(\text{Use } \pi = \frac{22}{7}\right)$

10. A wire of length 96 cm is bent so as to form an arc of a circle of radius 180 cm. What is the angle subtended at the centre in degrees?

11. If the arcs of the same lengths in two circles subtend angles 75° and 140° at the centre, then find the ratio of their radii.

12. Find the area of a sector whose arc length is 25π cm and angle of the sector is 60°.

13. The measures of angles of a quadrilateral arc in the ratio 2 : 5 : 8 : 9. Find their measures in degrees and in radians.

14. The angles of a triangle are in A.P. and the greatest angle is 100°. Find all the three angles in radians.

15. Find the angle between the hour-hand and minute-hand of a clock at
   i. thirty minutes past eight
   ii. quarter past one

16. Find the degree and radian measure of the angle between the hour-hand and the minute-hand of a clock at thirty minutes past three.

17. A horse is tied to a post by a rope. If the horse moves along a circular path always keeping the rope tight and describes 88 metres when it traces an angle of 72° at the centre, find the length of the rope. $\left(\text{Use } \pi = \frac{22}{7}\right)$

18. In a circle of diameter 66 cm, the length of a chord is 33 cm. Find the length of minor arc of the chord.

19. The angles of a quadrilateral are in A.P. and the greatest angle is five times the least. Express the least angle in radians.

Multiple Choice Questions

1. The angle subtended at the centre of a circle of radius 3 metres by an arc of length 1 metre is equal to
(A) 20° (B) 60°
(C) $\frac{1}{3}$ radian (D) 3 radians

2. A circular wire of radius 7 cm is cut and bend again into an arc of a circle of radius 12 cm. The angle subtended by the arc at the centre is
(A) 50° (B) 210°
(C) 100° (D) 60°

3. The radius of the circle whose arc of length 15 cm makes an angle of $\frac{3}{4}$ radian at the centre is
(A) 10 cm (B) 20 cm
(C) $11\frac{1}{4}$ cm (D) $22\frac{1}{2}$ cm

4. Convert $\frac{4\pi^c}{5}$ into degrees
(A) 144° (B) 60°
(C) 120° (D) 135°

5. Convert $\frac{8\pi^c}{3}$ into degrees
(A) 144° (B) 80°
(C) 480° (D) 180°

6. Convert 36° into radians
(A) $\frac{\pi^c}{6}$ (B) $\frac{\pi^c}{5}$
(C) $\frac{\pi^c}{3}$ (D) $\frac{\pi^c}{2}$

7. Convert –520° into radians
(A) $\frac{24}{9}\pi^c$ (B) $\frac{25}{9}\pi^c$
(C) $\frac{23}{9}\pi^c$ (D) $\frac{-26}{9}\pi^c$

8. The angles of a triangle are in A. P. such that greatest is 5 times the least. The angles in degrees are
(A) 30°, 60°, 100° (B) 30°, 45°, 90°
(C) 20°, 45°, 180° (D) 20°, 60°, 100°

9. The angles of a quadrilateral are in the ratio 2 : 3 : 3 : 4. Then the least angle in degrees is
(A) 90° (B) 45°
(C) 30° (D) 60°

10. The angles of a triangle are in the ratio 3 : 7 : 8. Then the greatest angle in radians is
(A) $\frac{4\pi^c}{9}$ (B) $\frac{5\pi^c}{9}$
(C) $\frac{7\pi^c}{18}$ (D) $\frac{\pi^c}{6}$

11. The difference between two acute angles of a right angled triangle is $\frac{\pi}{9}$. Then the angles in degrees are
(A) 30°, 35° (B) 45°, 55°
(C) 55°, 35° (D) 60°, 75°

12. Angle between the hour hand and minute hand of a clock at quarter past eleven in degrees is
(A) $\left(\frac{15\pi}{24}\right)^c$ (B) 112°, 30′
(C) 107°, 73″ (D) $\left(\frac{2\pi}{3}\right)^c$

13. The interior angles of a regular polygon of 15 sides in radians is
(A) $\frac{13\pi}{15}^c$ (B) $\frac{9\pi}{20}^c$
(C) 156° (D) 135°

14. The arc length of a circle is
(A) $s = r\theta$ (B) $S = \frac{\theta}{\pi}$
(C) $\frac{1}{2}r^2\theta$ (D) $\pi\theta$

15. The length of arc of a circle of radius 9 cm; subtending an angle of 40° at the centre is
(A) 2π cm (B) 12π cm
(C) $\frac{2\pi}{9}$ cm (D) $\frac{4\pi}{5}$ cm

16. OA and OB are two radii of a circle of radius 10 such that m∠AOB = 144°. Then area of the sector AOB is
(A) 8π sq.cm. (B) 20π sq.cm.
(C) 30π sq.cm. (D) 40π sq.cm.

17. The perimeter of a sector of a circle of area 36π sq. cm is 24 cm. Then the area of sector is
(A) 40 sq.cm. (B) 36 sq.cm.
(C) 46 sq.cm. (D) 26 sq.cm.

18. A semicircle is divided into two sectors, whose angles are in the ratio1: 2. Then the ratio of their area is
(A) 1:3 (B) 1:4
(C) 2:3 (D) 1:2

19. If θ^c is the angle between two radii of a circle of radius r, then the area of corresponding sector is
(A) $r^2\theta$ (B) $\frac{1}{2}r^2\theta$
(C) $r\theta$ (D) $2\pi r$

20. A wire 121 cm. long is bent so as to lie along the arc of a circle of 180 cm radius. The angle subtended at the centre of the arc in degrees is
(A) 35°, 37′ (B) 36°, 30′
(C) 37°, 30′ (D) 38°, 30′

Answers to Additional Practice Problems

Based on Exercise 1.1

1. i. coterminal
ii. not coterminal

2. i. $(112.5)^\circ$ ii. -150°
iii. $\left(\frac{1080}{\pi}\right)^\circ$ iv. $\left(\frac{45}{\pi}\right)^\circ$
v. 63°

3. i. $\left(\frac{5\pi}{6}\right)^c$ ii. $\left(\frac{17\pi}{9}\right)^c$
iii. $\left(-\frac{5\pi}{4}\right)^c$ iv. $\left(-\frac{\pi}{720}\right)^c$

4. i. 125° 18′ ii. 50° 54′
iii. 39°22′30″

5. $\theta = -40, \phi = \frac{5\pi}{2}$

6. $m\angle R = 60^\circ = \left(\frac{\pi}{3}\right)^c$

7. 55°, 35°

8. 290°, 250°

9. i. 24°, 72°, 84°
ii. $\left(\frac{2\pi}{9}\right)^c, \left(\frac{\pi}{3}\right)^c, \left(\frac{5\pi}{9}\right)^c, \left(\frac{8\pi}{9}\right)^c$.

10. 80°, 100°, 120°; $\left(\frac{4\pi}{9}\right)^c, \left(\frac{5\pi}{9}\right)^c, \left(\frac{2\pi}{3}\right)^c$.

Based on Exercise 1.2

1. 2π cm
2. 4π cm
3. $\frac{7\pi}{3}$ cm
4. $\frac{16\pi}{5}$ cm
5. 4 : 3
6. $\frac{3\pi}{2}$ sq. cm
7. $\frac{16\pi}{3}$ cm, $\frac{32\pi}{3}$ sq.cm
8. 2^c
9. $12\left(2\pi - 3\sqrt{3}\right)$sq.cm
10. 105 sq. cm.

Based on Miscellaneous Exercise - 1

1. i. $\left(\frac{\pi}{32}\right)^c$ ii. $\left(-\frac{1427\pi}{7200}\right)^c$

2. i. 83° 6′ 58″ (approx.)
ii. 17° 46″ (approx.)

3. $12\pi^c$

4. $m\angle Y = 45^\circ = \left(\frac{\pi}{4}\right)^c$

5. $\left(\frac{5\pi}{7}\right)^c$

6. 6.

7. $\frac{25}{2}(\pi - 2)$ sq. cm.

8. π cm.

9. 27.3 cm

10. $\left(\frac{96}{\pi}\right)^\circ$

11. 28 : 15

12. 937.5 π sq. cm.

13. 30°, 75°, 120°, 135°; $\left(\frac{\pi}{6}\right)^c, \left(\frac{5\pi}{12}\right)^c, \left(\frac{2\pi}{3}\right)^c,$
$\left(\frac{3\pi}{4}\right)^c$

14. $\left(\frac{\pi}{9}\right)^c, \left(\frac{\pi}{3}\right)^c, \left(\frac{5\pi}{9}\right)^c$

15. i. 75°
ii. $\left(52\frac{1}{2}\right)^o$

16. 75°, $\left(\frac{5\pi}{12}\right)^c$

17. 70 m

18. 11π cm

19. $\left(\frac{\pi}{6}\right)^c$

Answers to Multiple Choice Questions

1. (C)	2. (B)	3. (B)	4. (A)
5. (C)	6. (B)	7. (D)	8. (D)
9. (D)	10. (A)	11. (C)	12. (B)
13. (A)	14. (A)	15. (A)	16. (D)
17. (B)	18. (D)	19. (B)	20. (D)