Written as per the revised syllabus prescribed by the Maharashtra State Board of Secondary and Higher Secondary Education, Pune.

Perfect Mathematics – I

STD. XII Sci. & Arts

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01 Mathematical Logic

Subtopics

1.1 Statement

1.2 Logical Connectives, Compound Statements and Truth Tables

1.3 Statement Pattern and Logical Equivalence Tautology, Contradiction and Contingency

1.4 Quantifiers and Quantified Statements

1.5 Duality

1.6 Negation of Compound Statement

1.7 Algebra of Statements (Some Standard equivalent Statements)

1.8 Application of Logic to Switching Circuits

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Using Quantifiers Convert Open sentences into True statement	1.6	Q.2
Prepare the Truth Table/Find Truth Values of p and q for given cases	1.4	Q.7
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Algebra of statements (without using Truth Table verify the Logical Equivalence)/Rewrite the statement without using the conditional form	1.8	Q.3
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Introduction

Mathematics is an exact science. Every mathematical statement must be precise. Hence, there has to be proper reasoning in every mathematical proof.

Proper reasoning involves logic. The study of logic helps in increasing one's ability of systematic and logical reasoning. It also helps to develop the skills of understanding various statements and their validity.

Logic has a wide scale application in circuit designing, computer programming etc. Hence, the study of logic becomes essential.

Statement and its truth value

There are various means of communication viz., verbal, written etc. Most of the communication involves the use of language whereby, the ideas are conveyed through sentences.

There are various types of sentences such as:

i. Declarative (Assertive)
ii. Imperative (A command or a request)
iii. Exclamatory (Emotions, excitement)
iv. Interrogative (Question)

Statement

A statement is a declarative sentence which is either true or false but not both simultaneously. Statements are denoted by the letters p, q, r….

For example:

i. 3 is an odd number.
ii. 5 is a perfect square.
iii. Sun rises in the east.
iv. $x + 3 = 6$, when $x = 3$.

Truth Value

A statement is either True or False. The Truth value of a 'true' statement is defined to be T (TRUE) and that of a 'false' statement is defined to be F (FALSE).

Note: 0 and 1 can also be used for T and F respectively.

Consider the following statements:

i. There is no prime number between 23 and 29.
ii. The Sun rises in the west.
iii. The square of a real number is negative.
iv. The sum of the angles of a plane triangle is 180°.

Here, the truth value of statement i. and iv. is T and that of ii. and iii. is F.

Note:

The sentences like exclamatory, interrogative, imperative etc., are not considered as statements as the truth value for these statements cannot be determined.

Open sentence

An open sentence is a sentence whose truth can vary according to some conditions, which are not stated in the sentence.

Note:

Open sentence is not considered as statement in logic.

For example:

i. $x \times 5 = 20$
This is an open sentence as its truth depends on value of x (if $x = 4$, it is true and if $x \neq 4$, it is false).

ii. Chinese food is very tasty.
This is an open sentence as its truth varies from individual to individual.

Exercise 1.1

State which of the following sentences are statements. Justify your answer. In case of the statements, write down the truth value.

[1 Mark each]

i. The Sun is a star.
ii. May God bless you!
iii. The sum of interior angles of a triangle is 180°.
iv. Every real number is a complex number.
v. Why are you upset?
vi. Every quadratic equation has two real roots.
vii. $\sqrt{-9}$ is a rational number.
viii. $x^2 - 3x + 2 = 0$, implies that $x = -1$ or $x = -2$.
ix. The sum of cube roots of unity is one.
x. Please get me a glass of water.
xi. He is a good person.
xii. Two is the only even prime number.
xiii. $\sin 2\theta = 2\sin\theta\cos\theta$ for all $\theta \in R$.
xiv. What a horrible sight it was!
xv. Do not disturb.
xvi. $x^2 - 3x - 4 = 0$, $x = -1$.
xvii. Can you speak in French?
xviii. The square of every real number is positive.
xix. It is red in colour.
xx. Every parallelogram is a rhombus.

***Solution*:**

i. It is a statement which is true, hence its truth value is ‘T’.
ii. It is an exclamatory sentence, hence, it is not a statement.
iii. It is a statement which is true, hence its truth value is ‘T’.
iv. It is a statement which is true, hence its truth value is ‘T’.
v. It is an interrogative sentence, hence it is not a statement.
vi. It is a statement which is false, hence its truth value is ‘F’.
vii. It is a statement which is false, hence its truth value is ‘F’.
viii. It is a statement which is false, hence its truth value is ‘F’.
ix. It is a statement which is false, hence its truth value is ‘F’.
x. It is an imperative sentence, hence it is not a statement.
xi. It is an open sentence, hence it is not a statement.
xii. It is a statement which is true, hence its truth value is ‘T’.
xiii. It is a statement which is true, hence its truth value is ‘T’.
xiv. It is an exclamatory sentence, hence it is not a statement.
xv. It is an imperative sentence, hence it is not a statement.
xvi. It is a statement which is true, hence its truth value is ‘T’.
xvii. It is an interrogative sentence, hence, it is not a statement.
xviii. It is a statement which is false, hence its truth value is ‘F’. (Since, 0 is a real number and square of 0 is 0 which is neither positive nor negative).
xix. It is an open sentence, hence it is not a statement. (The truth of this sentence depends upon the reference for the pronoun ‘It’.)
xx. It is a statement which is false, hence its truth value is ‘F’.

Logical Connectives, Compound Statements and Truth Tables

Logical Connectives:
The words or group of words such as “and, or, if …. then, if and only if, not” are used to join or connect two or more simple sentences. These connecting words are called logical connectives.

Compound Statements:
The new statement that is formed by combining two or more simple statements by using logical connectives are called compound statements.

Component Statements:
The simple statements that are joined using logical connectives are called component statements.

For example:
Consider the following simple statements,
i. e is a vowel
ii. b is a consonant

These two component statements can be joined by using the logical connective ‘or’ as shown below:
‘e is a vowel or b is a consonant’
The above statement is called **compound statement** formed by using logical connective **‘or’.**

Truth Table

A table that shows the relationship between truth values of simple statements and the truth values of compounds statements formed by using these simple statements is called truth table.

Note:
The truth value of a compoud statement depends upon the truth values of its component statements.

Logical Connectives

A. AND [$\wedge$] (Conjunction):
If p and q are any two statements connected by the word ‘and’, then the resulting compound statement ‘p and q’ is called conjunction of p and q which is written in the symbolic form as ‘$p \wedge q$’.

For example:
p: Today is a pleasant day.
q: I want to go for shopping.
The conjunction of above two statements is ‘$p \wedge q$’ i.e. ‘Today is a pleasant day and I want to go for shopping’.
A conjunction is true if and only if both p and q are true.

Truth table for conjunction of p and q is as shown below:

p	q	$p \wedge q$
T	**T**	**T**
T	F	F
F	T	F
F	F	F

Note:
The words such as but, yet, still, inspite, though, moreover are also used to connect the simple statements.
These words are generally used by replacing 'and'.

B. OR [$\vee$] (Disjunction):

If p and q are any two statements connected by the word 'or', then the resulting compound statement 'p or q' is called disjunction of p and q which is written in the symbolic form as '$p \vee q$'.

The word 'or' is used in English language in two distinct senses, exclusive and inclusive.

For example:

i. Rahul will pass or fail in the exam.
ii. Candidate must be graduate or post-graduate.

In eg. (i), 'or' indicates that only one of the two possibilities exists but not both which is called exclusive sense of 'or'. In eg. (ii), 'or' indicates that first or second or both the possibilities may exist which is called inclusive sense of 'or'.

A disjunction is false only when both p and q are false.

Truth table for disjunction of p and q is as shown below:

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	**F**	**F**

Exercise 1.2

1. Express the following statements in symbolic form: ***[1 Mark each]***

i. Mango is a fruit but potato is a vegetable.
ii. Either we play football or go for cycling.
iii. Milk is white or grass is green.
iv. Inspite of physical disability, Rahul stood first in the class.
v. Jagdish stays at home while Shrijeet and Shalmali go for a movie.

***Solution*:**

i. Let p : Mango is a fruit, q : Potato is a vegetable.
∴ The symbolic form of the given statement is $p \wedge q$.

ii. Let p : We play football, q : We go for cycling.
∴ The symbolic form of the given statement is $p \vee q$.

iii. Let p : Milk is white, q : Grass is green.
∴ The symbolic form of the given statement is $p \vee q$.

iv. Let p : Rahul has physical disability,
q : Rahul stood first in the class.
The given statement can be considered as 'Rahul has physical disability and he stood first in the class.'
∴ The symbolic form of the given statement is $p \wedge q$.

v. Let p : Jagdish stays at home,
q : Shrijeet and Shalmali go for a movie.
The given statement can be considered as 'Jagdish stays at home and Shrijeet and Shalmali go for a movie.'
∴ The symbolic form of the given statement is $p \wedge q$.

2. Write the truth values of following statements. ***[1 Mark each]***

i. $\sqrt{3}$ is a rational number or 3 + i is a complex number.
ii. Jupiter is a planet and Mars is a star.
iii. $2 + 3 \neq 5$ or $2 \times 3 < 5$
iv. $2 \times 0 = 2$ and $2 + 0 = 2$
v. 9 is a perfect square but 11 is a prime number.
vi. Moscow is in Russia or London is in France.

***Solution*:**

i. Let p : $\sqrt{3}$ is a rational number,
q : 3 + i is a complex number.
∴ The symbolic form of the given statement is $p \vee q$.
Since, truth value of p is F and that of q is T.
∴ truth value of $p \vee q$ is T

ii. Let p : Jupiter is a planet,
q : Mars is a star.
∴ The symbolic form of the given statement is $p \wedge q$.
Since, truth value of p is T and that of q is F.
∴ truth value of $p \wedge q$ is F

iii. Let $p : 2 + 3 \neq 5$,
$q : 2 \times 3 < 5$.

∴ The symbolic form of the given statement is $p \vee q$.

Since, truth value of both p and q is F.

∴ truth value of $p \vee q$ is F

iv. Let $p : 2 \times 0 = 2$,
$q : 2 + 0 = 2$.

∴ The symbolic form of the given statement is $p \wedge q$.

Since, truth value of p is F and that of q is T.

∴ truth value of $p \wedge q$ is F

v. Let p : 9 is a perfect square,
q : 11 is a prime number.

∴ The symbolic form of the given statement is $p \wedge q$.

Since, truth value of both p and q is T.

∴ truth value of $p \wedge q$ is T

vi. Let p : Moscow is in Russia,
q : London is in France.

∴ The symbolic form of the given statement is $p \vee q$.

Since, truth value of p is T and that of q is F.

∴ truth value of $p \vee q$ is T

C. Not [~] (Negation):

If p is any statement then negation of p i.e., 'not p' is denoted by ~p. Negation of any simple statement p can also be formed by writing 'It is not true that' or 'It is false that', before p.

For example:

p : Mango is a fruit.

~p : Mango is not a fruit.

Truth table for negation is as shown below:

p	~p
T	F
F	T

Note: If a statement is true its negation is false and vice-versa.

Exercise 1.3

Write negations of the following statements:

i. Rome is in Italy.

ii. 5 + 5 = 10

iii. 3 is greater than 4.

iv. John is good in river rafting.

v. π is an irrational number.

vi. The square of a real number is positive.

vii. Zero is not a complex number.

viii. Re (z) ≤ | z |.

ix. The sun sets in the East.

x. It is not true that the mangoes are inexpensive.

***Solution*:**

i. Rome is not in Italy.

ii. $5 + 5 \neq 10$

iii. 3 is not greater than 4.

iv. John is not good in river rafting.

v. π is not an irrational number.

vi. The square of a real number is not positive.

vii. Zero is a complex number.

viii. Re (z) > |z|.

ix. The sun does not set in the East.

x. It is true that the mangoes are inexpensive.

D. If….then (Implication, ⟶) (Conditional):

If p and q are any two simple statements, then the compound statement, 'if p then q', meaning "statement p implies statement q or statement q is implied by statement p", is called a conditional statement and is denoted by $p \to q$ or $p \Rightarrow q$.

Here p is called the antecedent (hypothesis) and q is called the consequent (conclusion).

For example:

Let p: I travel by train.
q: My journey will be cheaper.

Here the conditional statement is

'$p \to q$: If I travel by train then my journey will be cheaper.'

Conditional statement is false if and only if antecedent is true and consequent is false.

Truth table for conditional is as shown below:

p	q	$p \to q$
T	T	T
T	**F**	**F**
F	T	T
F	F	T

Note: Equivalent forms of the conditional statement

$p \to q$:

a. p is sufficient for q.

b. q is necessary for p.

c. p implies q.

d. p only if q.

e. q follows from p.